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Hyperacusis: A Graph Theoretical Model

Hassan Ibrahim*, Amechi Innocent Ezike & Vanenchii Petter Ayoo
Department of Mathematics, Federal University of Lafia, PMB 146, Nasarawa State, Nigeria

Abstract

Hyperacusis, characterized by increased sensitivity to sounds, presents growing challenges for effective diagnosis and intervention. This study utilizes graph theoretical metrics, specifically the calculation of the metric dimension of a graph to visualize and quantify the intricate network of interactions within the etiological landscape of Hyperacusis and it's potential causes. By constructing a graph $H = (V,E)$, where nodes (V) represent factors such as aging, trauma, infections, depression, and migraines, and edges (E) model their interactions, we calculate the metric dimension (β (H)) to identify critical factors. To further address the inherent complexity of calculating the resolving set for such a complex graph, the shortest path algorithm was utilized to determine the minimal resolving set, enhancing the precision and efficiency of the metric dimension analysis. This network-centric approach provides insights into the importance and interdependencies of these causes, reducing the complexity from thirteen possible factors to six key contributors. These findings offer a framework for targeted clinical interventions, contributing to the broader understanding of Hyperacusis etiology and improving patient management.

Keywords: Hyperacusis, netric dimension, resolving sets, shortest path

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***Correspondences**
H. Ibrahim ✉
hifulafia@gmail.com

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Introduction

Hyperacusis, often described as an overwhelming sensitivity to everyday sounds, remains a condition clouded by limited understanding and increasing complexity. For individuals living with Hyperacusis, ordinary auditory experiences become sources of discomfort and distress, profoundly affecting their quality of life. Despite growing recognition within the medical community, the condition continues to challenge researchers and clinicians alike, with its multifactorial nature complicating both diagnosis and treatment. In a foundational study by Richard Tyler [1], a bar chart highlighted 14 potential causes of Hyperacusis, ranging from physiological factors such as aging and hearing loss to environmental triggers like noise exposure and infections. While these findings provided a critical starting point, the inclusion of a generalized "other causes" category underscored the need for further clarity. In this study, we refine this framework by modeling 13 clearly defined causes as vertices in a graph, omitting the 'other causes' category for precision. Using graph theory, a mathematical approach renowned for its capacity to represent and analyze complex networks, we explore the intricate interplay of these causes. Metric dimension has been widely studied in graph theory for its applications in network design, robot navigation, and disease spread modeling [2]. The metric dimension is a crucial graph invariant that helps us understand how uniquely we can identify the vertices in a network. Chaudbury *et al.* [3] dive into the metric dimension of various lattice networks, focusing on Bakelite, Backbone DNA, and Polythiophene networks. The application of metric dimension of graphs in neuroscience, human disease, and drug developments from the perspectives of network science are discussed in biological models and data applications, [4], protein-protein interaction of Autism patients [5], GeSbTe supper lattice chemical structure [6], Gene regulatory network dynamics [7], circumcoronene series of benzenoid networks [8], and many others.

Hyperacusis has many known causes and associations, although most cases have no known cause. There are a few diseases and syndromes that are associated with Hyperacusis, for example, migraine, depression, posttraumatic stress disorder, head injury, Lyme disease, Williams syndrome, fibromyalgia, Addison's disease, autism, myasthenia gravis, and middle cerebral aneurysm. Like hearing loss and tinnitus, Hyperacusis probably can be associated with both peripheral and central factors. Hyperacusis is often accompanied by a cochlear hearing loss (although we discuss below how this might be overemphasized), and this usually involves damage to cochlear hair cells and subsequent auditory nerve degeneration. However, annoyance, fear, and pain Hyperacusis must involve central mechanisms. An often cited theory of Hyperacusis is that the central auditory system turns up a "central gain" to compensate for peripheral hearing loss. Hyperacusis and tinnitus are often related and the estimates of the prevalence of tinnitus in Hyperacusis patients include 86, 60 and 40% [9-11]. Other studies have reported estimates ranging from 40 to 79% [12, 13]. However, Ulf *et al*. [14] found that only 21% (Internet sample) and 9% (postal sample) of people reporting hyperacusis also reported tinnitus. The variation in the prevalence of tinnitus with Hyperacusis across studies is influenced by different definitions and criteria for diagnosing Hyperacusis and tinnitus. It should also be noted that much of the literature on tinnitus and Hyperacusis comes from tinnitus clinics and might not be representative of the general population. Thus, the reports on Hyperacusis are more likely to be populations with tinnitus (and with hearing loss). There might very well be a large population of people with Hyperacusis but without tinnitus or without hearing loss. The prevalence of Hyperacusis in those with tinnitus may be higher than in the population at large. It is also important to recognize that loud noise can make tinnitus worse in some tinnitus sufferers. This might be confused with loudness Hyperacusis. There is a paucity of drug trials for Hyperacusis. As for tinnitus, most people with Hyperacusis would prefer a pill to cure them [15]. Physiological and behavioral animal models will help define the mechanisms and therefore provides some direction for medical interventions. However, there are many forms of Hyperacusis involving complex behaviors, and it will be difficult and require much time to develop valid animal models for all forms. Perhaps it is comforting to note that there are effective drug therapies for many maladies, such as mental illness, in which the underlying mechanisms are not completely understood. Additionally, drug trials should focus on individuals, not groups.

Preliminaries

Resolving set in a graph: For an undirected graph G, a resolving set R is a subset of vertices $R \subseteq V(G)$ such that every pair of distinct vertices $u, v \in V(G)$ can be distinguished by the set of their distances to the vertices in R. Formally, for every pair $u, v \in V(G)$, there exists a vertex $r \in R$ such that $d(u,r) \neq d(v,r)$, where $d(x,y)$ denotes the distance between vertices x and y in G.

Metric dimension of a graph: For an undirected graph G, the metric dimension $\beta(G)$ is the minimum cardinality of a subset $S \subseteq V(G)$ such that, for every pair of vertices $u, v \in V(G)$, there exists a vertex $s \in S$ such that $d(u,s) \neq d(v,s)$, where $d(x,y)$ denotes the distance between vertices x and y in G.

The Floyd-Warshall Algorithm: The Floyd-Warshall algorithm can also be used to obtain the resolving set of a graph which in turn expose the easy layout of the metric dimension through computing the shortest paths between all pairs of vertices in a graph. It works as follows:

1. **Initialization**
   - Let $G = (V, E)$ be the graph with n vertices.
   - Create a distance matrix D such that:
   $$D[i][j] = \begin{cases} w(i,j) & \text{if}(i,j) \in E, \\ 0 & \text{if} i = j, \\ \infty & \text{otherwise}, \end{cases}$$
   where $w(i,j)$ is the weight of the edge from i to j.

2. **Algorithm iteration:** For each vertex $k \in V$:
   - Update the distance matrix D as follows:
   $$D[i][j] = \min(D[i][j], D[i][k] + D[k][j]),$$
   for all pairs of vertices $i, j \in V$.

3. **Final output:** After all iterations, the matrix $D[i][j]$ contains the shortest distance from vertex i to vertex j.

Definition 3: *Let $G = (V, E)$ be a graph where V is the set of vertices and E is the set of edges. The **shortest path** between two vertices $u, v \in V$ is a sequence of vertices $P = \langle v_0, v_1, \dots, v_k \rangle$ such that:*

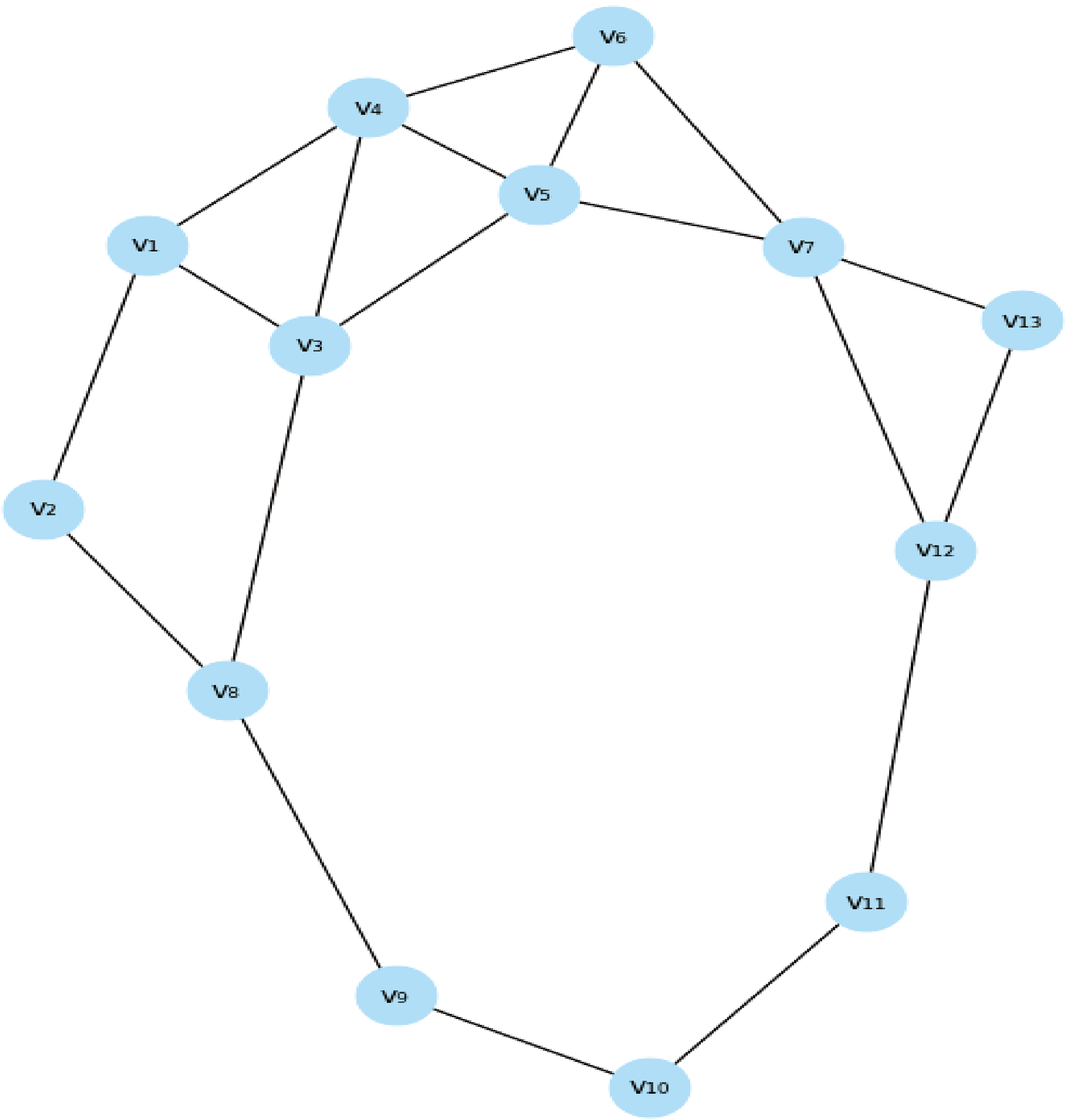
1. $v_0 = u$ and $v_k = v$,
2. $\{v_i, v_{i+1}\} \in E$ for all $i = 0,1, \dots, k-1$,
3. The length k (i.e., the number of edges in P) is minimized among all possible paths from u to v.

If the graph G is **weighted**, with a weight function $w: E \to \mathbb{R}_{\geq 0}$, the shortest path between u and v is the path P that minimizes the total weight:
$$P = \operatorname{argmin} \sum_{i=0}^{k-1} w(v_i, v_{i+1}),$$
where $w(v_i, v_{i+1})$ is the weight of the edge $\{v_i, v_{i+1}\}$.

Graph Construction of Hyperacusis Graph

The possible hyperacusis causes are the biological, clinical, and psychological factors: Noise exposure (continuous), Noise exposure (impulsive), Hearing noise (sudden), Anxiety, Medication, Hearing loss (long term), Meniere's disease, Ear infection, Aging, Head trauma, Infection/virus, Depression and Migraine [1].

To construct a hyperacusis graph, H, of possible hyperacusis causes, treat biological, clinical, and psychological factors as nodes, and plausible causal relations as edges. Thus, we have the below graph construction:



Where, v_1: Noise exposure (continuous); v_2: Noise exposure (impulsive); v_3: Hearing noise (sudden); v_4: Anxiety; v_5: Medication; v_6: Hearing loss (long term); v_7: Meniere's disease; v_8: Ear infection; v_9: Aging; v_{10}: Head trauma; v_{11}: Infection/virus; v_{12}: Depression; v_{13}: Migraine

Figure 1: Graph Representation of Hyperacusis Causes

We will use the shortest path approach in Table 1 below to investigate our resolving sets.

Result and Discussion

Theorem 1: If a vertex v appears in more than half of the minimal resolving sets of H, then v is critical for the resolving power of the graph.

Proof: Let $\mathcal{S}$ represent the collection of all minimal resolving sets of the graph H. Assume that each minimal resolving set S has a size of k, so $S \in \mathcal{S}$ implies $|S| = k$. Define $f(v)$ as the number of minimal resolving sets that include the vertex v. If $f(v) > \frac{|\mathcal{S}|}{2}$, then v appears in more than half of the minimal resolving sets of H. Removing v would result in many minimal resolving sets losing their resolving power, thereby making v critical to the resolving power of the graph. This implies that v plays a significant role in maintaining the metric dimension $\beta(H)$ of the graph.

Theorem 2: In a stable hyperacusis graph H, the minimal resolving sets exhibit a high degree of overlap

Proof: Let $S_1, S_2, \ldots, S_m$ be the minimal resolving sets. Calculate the overlap $O(S_i, S_j) = |S_i \cap S_j|$ for all pairs (i, j). If $O(S_i, S_j) > \frac{|S_i|}{2}$ for most pairs, then the graph is stable. High overlap suggests that specific vertices are consistently necessary for

resolving, highlighting strong interdependencies.
Using the Floyd-Warshall Algorithm, the shortest paths between all pairs of vertices in H was computed.

Initialization

Let the graph H = (V, E) have n =|V| vertices. Define d[*i*][*j*] as the shortest distance between vertex *i* and vertex *j*. The distances are initialized as follows:

$$d[i][j] = \begin{cases} w(i,j), & \text{if}(i,j) \in E(\text{edgeexists}), \\ 0, & \text{if} i = j(\text{distancetoselfiszero}), \\ \infty, & \text{otherwise}(\text{nodirectedgeexists}). \end{cases}$$

Iterative updates

For each intermediate vertex $k \in V$, update the distances d[*i*][*j*] for all pairs of vertices (*i*, *j*) using the relation:

$$d[i][j] = \min(d[i][j], d[i][k] + d[k][j]).$$

This ensures that the shortest path between i and j is accurately recorded, either directly or through the intermediate vertex *k*.

Completion

After iterating over all $k \in V$, the final table d[*i*][*j*] contains the shortest distances between every pair of vertices in H.

Table 1: Computation of the distance matrix (Shortest Path) of the Hyperacusis Graph, H

(i/j)	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9	v_{10}	v_{11}	v_{12}	v_{13}
v_1	0	1	1	1	2	2	3	2	3	4	5	4	4
v_2	1	0	2	2	3	3	4	1	2	3	4	5	5
v_3	1	2	0	1	1	2	2	1	2	3	4	3	3
v_4	1	2	1	0	1	1	2	2	3	4	4	3	3
v_5	2	3	1	1	0	1	1	2	3	4	3	2	2
v_6	2	3	2	1	1	0	1	3	4	5	3	2	2
v_7	3	4	2	2	1	1	0	3	4	3	2	1	1
v_8	2	1	1	2	2	3	3	0	1	2	3	4	4
v_9	3	2	2	3	3	4	4	1	0	1	2	3	4
v_{10}	4	3	3	4	4	4	3	2	1	0	1	2	3
v_{11}	5	4	4	4	3	3	2	3	2	1	0	1	2
v_{12}	4	5	3	3	2	2	1	4	3	2	1	0	1
v_{13}	5	5	3	3	2	2	1	4	4	3	2	1	0

This systematic approach ensures that all possible paths are considered, and the shortest-path table is obtained through a sequence of minimization operations combined with path length calculations. The result is a comprehensive representation of the shortest paths within the graph. By careful observation of the shortest path Table 1, we can deduce the least cardinal resolving sets of the model graph and these are extracted as $\{v2, v9\}$, $\{v6, v11\}$, $\{v6, v12\}$, $\{v6, v13\}$, $\{v9, v13\}$. From Definition 2, the above result shows that there exists a vertex $s \in S$ such that $d(u,s) \neq d(v,s)$, where $d(x,y)$ denotes the distance between vertices x and y in H. The metric representations in Table 2 that resolves the graph are the subset $s \in S$ whose distance are distinct shown as $v_2 \neq v_9$, $v_6 \neq v_{11}$, $v_6 \neq v_{12}$, $v_6 \neq v_{13}$, $v_9 \neq v_{13}$. Thus $\beta(H) = 2$.

The study of Hyperacusis, characterized by an increased sensitivity to normal environmental sounds, benefits significantly from the application of metric dimension analysis in graph theory. By representing the causes of Hyperacusis as vertices in a graph H, with edges denoting relationships between these causes, we can derive important insights into the structural properties and interdependencies of these causes. It was observed that the frequency of vertices appearing in minimal resolving sets provides a clear measure of their importance. In graph H, the minimal resolving sets are subsets of vertices that uniquely identify all other vertices based on their distance vectors. By calculating the metric dimension $\beta(H)$ and identifying all minimal resolving sets S of H, we can determine the frequency $f(v)$ of each vertex v appearing in these sets. The vertices with the highest $f(v)$ values are the most critical in uniquely identifying other vertices. This is because these vertices provide the most unique distance vectors, which are essential for the resolving capability of the graph. Hence, understanding the frequency distribution helps in pinpointing the most influential causes of Hyperacusis. Moreover, the criticality of vertices in maintaining the resolving power of the graph is further highlighted by their presence in the minimal resolving sets. If a vertex v appears in more than half of these sets, it is deemed critical for the graph's resolving power. Removing such a vertex would result in many minimal resolving sets losing their effectiveness, thereby compromising the graph's metric dimension. This observation underscores the significance of certain vertices (or causes) that play a pivotal role in the overall structure of the Hyperacusis cause graph.

This study represents a significant step forward in the quantitative analysis of Hyperacusis, by reducing the thirteen (13) possible causes to six (6) central causes with v_6 taking the highest position of impact by repeating it appearance three (3) times. The next with also a high contribution to the disease is v_9 and v_{13} appearing twice each followed by v_2, v_9, v_{11}. The identification of long-term hearing loss, aging, depression, infection/virus, noise exposure and migraine as central causes suggests that these factors play a crucial role in the Hyperacusis landscape. The high frequency of these causes in resolving sets indicates their significant impact and interrelationships, highlighting the importance of addressing them in both clinical and public health contexts. These findings have significant implications for medical practice and research. Personalized medicine approaches can be developed to tailor treatments to individual patients based on their risk factors. Overall, this study provides a novel framework for understanding and addressing Hyperacusis, with the potential to significantly advance both theoretical research and practical clinical solutions.

Conclusion

This study comprehensively demonstrates that applying metric dimension analysis to a graph of etiological factors in hyperacusis can efficiently highlight a minimal set of six critical contributors from a larger pool of thirteen, offering a structured framework to prioritize diagnostic focus and guide more targeted, network-informed clinical interventions in the management of hyperacusis.

Conflict of interest: The authors have declared that there is no conflict of interest reported in this study.

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