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Dynamical System Analysis of Reference Evapotranspiration Using Chaos Models in some Selected States in Northern Nigeria

C. O. Iyamu[1], L. W. Lumbi[1], I. I. Ewa[1]* & E. V. Tikyaa[2]
[1]Department of Physics, Nasarawa State University, Keffi, Nigeria
[2]Department of Physics, Joseph Sarwuan Tarka University, Makurdi, Benue State, Nigeria

Abstract

This study analyzed the dynamical system of reference evapotranspiration using chaos and maximum entropy models in some selected states in Northern Nigeria. The study covered 16 states in Northern Nigeria which were selected using purposive sampling techniques. The materials used for the study consist of secondary data of daily mean records of air temperature, wind speed, solar radiation and relative humidity from 1st January 1984 to December 2022. From the data, the daily references Evapotranspiration were computed using Penman-Monteith equation. The reference evapotranspiration obtained was further analyzed using Phase portrait, recurrent plot and Lyapunov exponent to characterize the dynamical system. The results showed that the dynamical system has a random and nonlinear trend which is aperiodic. The phase portrait exhibits sponge-like structures having distinct curves which indicate a low degree deterministic chaos. Also, the recurrent plots in 16 states had a dark short diagonal line diagonals and white ribbons which indicates seasonal variations. The largest Lyapunov exponents of the 16 states are all positive and range between 0.001983/day – 0.004419/day. This positive result indicated that the dynamical system is deterministic and chaotic in behavior.

Keywords: Dynamical system, evapotranspiration, choas, lyapunov exponents, seasonal variation

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Corresponding author
I. I. Ewa ✉
ewainusa@nsuk.edu.ng

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Introduction

In physics, Evapotranspiration is the directional change of water to vapor and its movement to the atmosphere [1, 2]. This movement of water occurs from the soil in form of evaporation and from the plants in form of transpiration. The two process are combined to form evapotranspiration because they occur simultaneously.

Evapotranspiration is considered when calculating hydrologic water balance, distribution of water system, establishing irrigation system and assessing climate change problem [3]. In northern Nigeria, the major problem of climate change is aridity because it affects crop yield due to insufficient rainfall. Climate variation is of great concern and is a topic of interest in today's research trend [4]. Climate change is viewed to change water cycle and evapotranspiration globally [5]. Also, there is reasonable evidence to suggest that the water cycle has been changing with climate change [6]. Climate change which occurs because of temperature rise affects vapor pressure deficit, sunshine duration, evapotranspiration, precipitation, wind speed and relative humidity.

Evapotranspiration can be measured using Lysimeter and reference evapotranspiration equations but the use of Lysimeter is very difficult because it takes a long time and involves large areas. As a result of this, it makes the research work to be very expensive and time consuming. This makes the institution for Food and Agriculture Organization (FAO) recommend the use of localized areas instead of large areas to estimate evapotranspiration and then termed it as Reference Evapotranspiration (ETo) [7].

ETo is defined as the frequency of evapotranspiration from a localized area that have a crop height of 0.12 m, a crop surface resistance factor of 70 secm^{-1}, and an albedo factor of 0.23 [7]. They further said that localized crop consists of an extensive surface of green grass cover that have the same height. ETo can be computed from several ETo equations using recorded weather variables such as radiation, air temperature, air humidity and wind speed among others [8]. These ETo equations consist of Penman-Monteith equation, Priestley-Taylor equation, Hargreave-Samani equation, Adjusted Hargreaves equation etc. However, the most used equation is the Penman-Monteith equation [9]. They further explained that Penman-Monteith equation did not need local calibration and can be used to provides ETo values for a uniform grass reference surface worldwide.

The computation of ETo is further analyzed to determine the behavior of ETo using chaos models such as phase portraits, recurrent plots, lyapunov exponent and probability distribution curve. Tableau [10] remarked that characterizing ETo using chaos models can be used to make observations and predictions for future strategic decisions making. Chaos model considers that complex system having a long-time evolution may have chaotic features due to changes in conditions [11]. Also, it is a nonlinear system which exhibit apparent disorder or randomness in its behavior in the dynamical system [12].
Therefore, the dynamical system of ETo can best be used in nonlinear deterministic chaotic model which describes the changes that occur in the evapotranspiration system using it as a single variable in time series [13, 14, 15]. This approaches which is based on chaos theory are widely acceptable because it is possible to predict the future state of the system based on the use of daily and monthly weather data. Furthermore, behavior of chaotic system can be studied through techniques consisting of correlation dimension, recurrent plots, poincare maps etc [16].
Estimating and characterizing ETo using chaos model is highly required for the provision of weather information which can be used for water and irrigation management mostly in the arid region. It is in view of this that this study investigated the dynamical system of ETo using chaos modelsin some selected states in Northern part of Nigeria.

Model equation for reference evapotranspiration
The model equation used in computing reference evapotranspiration is Penman-Monteith equation. The formula for Penman-Monteith equation is given by:

$$ET_{o,pm} = \frac{0.408\Delta(R_n - G) + \gamma\frac{900}{T_k}U_2(e_s - e_a)}{\Delta + \gamma(1 + 0.34U_2)} \quad (mm/day) \qquad (1)$$

The parameters and formulae for Penman-Monteith equations consist of:
Mean air temperature, T:

Mean air temperature, $T = \frac{T_{min} + T_{max}}{2}$ (2)

Absolute temperature, $T_k = T + 273.16$ (3)

Psychrometric constant, γ [9]:

$$\gamma = \frac{C_p P}{\varepsilon\lambda} = 0.665 \times 10^{-3} \quad (kPa/°C) \qquad (4)$$

Where: C_p = specific heat capacity of surrounding air at constant pressure
$= 1.013 \times 10^3\ MJ/kg°C$
ε = ratio of molecular weight of water vapor to dry air = 0.622
λ = latent heat of vaporization = 2.45 MJ/k

Saturated water vapor pressure (e_s) [9]:

$$e_s(T) = 0.6108\ exp\left(\frac{17.27\ T}{T + 237.3}\right) \quad (kPa) \qquad (5)$$

T = average air temperature (°C)

Actual water vapor pressure (e_a) [9]:

$$e_a(T_{dew}) = 0.6108\ exp\left(\frac{17.27\ T_{dew}}{T_{dew} + 237.3}\right) \quad (kPa) \qquad (6)$$

Where; T_{dew} = dew point temperature which is computed from the relative humidity (RH) using the expression [17]:

$$T_{dew} = \left(\frac{RH}{100}\right)^{\frac{1}{8}}(112 + 0.9T) + 0.1T - 112 \qquad (7)$$

Vapor pressure deficit = $e_s - e_a$
Slope of saturated water vapor pressure (Δ)[9]:

$$\Delta = 4098\frac{e_s}{(T_k)^2} \qquad (8)$$

Net shortwave radiation, R_{ns}:

$$R_{ns} = (1 - \alpha)R_s(MJ/m^2day) \qquad (9)$$

R_s is the incident solar radiation and α is the albedo whose value is 0.23 [9].

Net long wave radiation, R_{nl}:
Net long wave radiation is defined as the net energy flux that comes out from the earth's surface. It is a function of the Stefan-Boltzmann's law with some correction from the relative humidity and cloud cover which acts as absorbers of longwave radiation and is expressed as [9]:

$$R_{nl} = \sigma\left[\frac{T^4_{max,k} + T^4_{min,k}}{2}\right]\left(0.34 - 0.14\sqrt{e_a}\right)\left(1.35\frac{R_s}{R_{so}} - 0.35MJm2day\right) \qquad (10)$$

Where σ = Stefan-Boltzmann's constant = $4.903 \times 10^{-9}\ MJK^{-4}m^{-2}day^{-1}$
Net Radiation, R_n:

$$R_n = R_{ns} - R_{nl} \qquad (11)$$

Wind speed relation:
The wind speed, $(U_2)$was determined at 2 m height above the ground [2]. Hence the height x is 2 m above the ground:

$$U_2 = \frac{4.87}{\ln(67.8x - 5.42)} \quad (km/hr) \qquad (12)$$

Soil heat flux density, G:
The soil heat flux compared to net radiation R_n is small. The reason is because the soil surface is covered by crop. Therefore, $G \approx 0$ [9].

Materials and Methods

The study covered Northern States in Nigeria. Northern Nigeria is located at latitudes 10° 30' 59.99" North of the equator and longitude 7° 25' 59.99" East of the Greenwich Meridian. It shared borders to the North with Niger, to the East with Chad and Cameroon and to the West with Benin Republic. In this study, 16 states from the 20 Northern states were selected using purposive sampling technique. The state capitals of these 16 states were investigated as reference locations and they consist of: of Makurdi, Dutse, Jos, Minna, Sokoto, Katsina, Kaduna, Bauchi, FCT-Abuja, Maiduguri, Jalingo, Yola, Gusau, Damaturu, Birnin-Kebbi and Kano.

Figure 1: Northern States in Nigeria

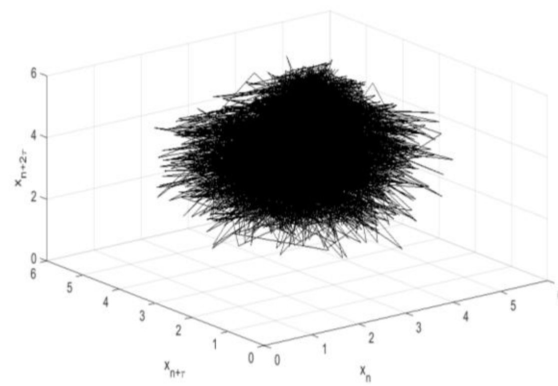
The materials used for analyses consist of secondary data obtained from Modern Era Retrospective Analysis for Research and Application (MERRA-2). The MERRA-2 weather record is sourced from Nigerian Meteorological Agency (NiMET) whose headquarter is located at Bill Clinton Drive, Airport Road, FCT-Abuja.These data consist of daily mean records of air temperature, wind speed, solar radiation and relative humidity for selected locations in Northern Nigeria from 1st January 1984 – 31st December 2022. The information from this data was inputted into MATLAB (R2020a) software after coding the Penman-Monteith equations formular and parameters into the software.

The result obtained from MATLAB (R2020a) gives the rate of reference evapotranspiration within the reference location. Also, MATLAB (R2020a) was also used in the analysis of phase portraits, recurrent plot and Lyapunov exponent.

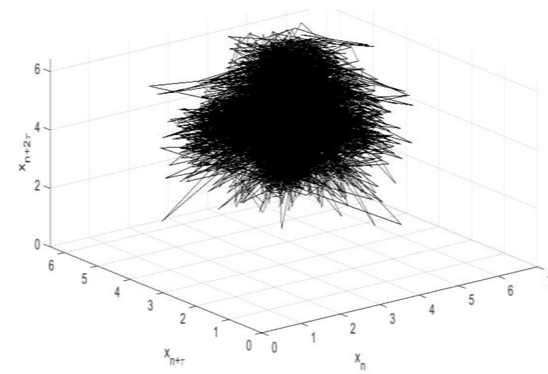
Results and Discussion

Phase portrait of reference evapotranspiration

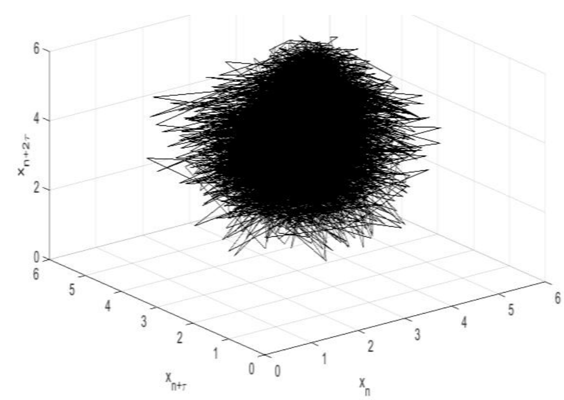
Phase portrait is a 3-dimensionalgeometrical representation of the same reference evapotranspiration (ETo) in each location of the 16 Northern states in a phase plane plot. The geometrical computation consists of calculating the number of the same ETo denoted as n across each of the 16 Northern states and their time lag denoted as τ. From these computations, the variables consisting of n, n+ τ and n + 2τ was computed. Then the values of the geometrical computations are fitted together in other to study the dynamical system of the motion of ETo in a phase plane. Based on these variables, a three-dimensional phase portrait was plotted for each of the 16 Northern states and displayed in Fig. 2.



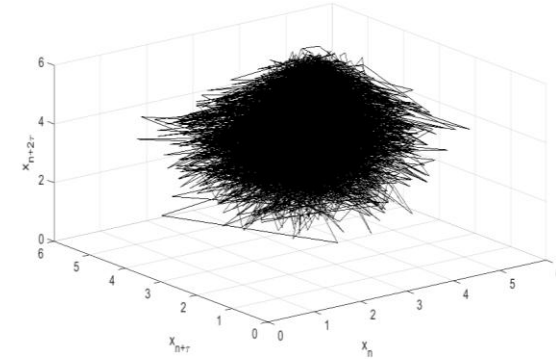
(a) Makurdi



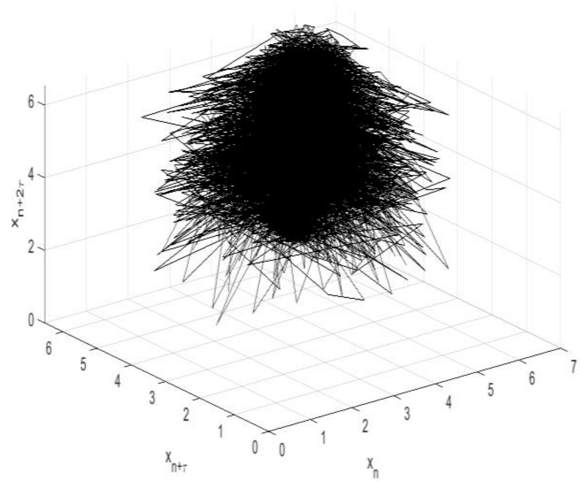
(b) Dutse



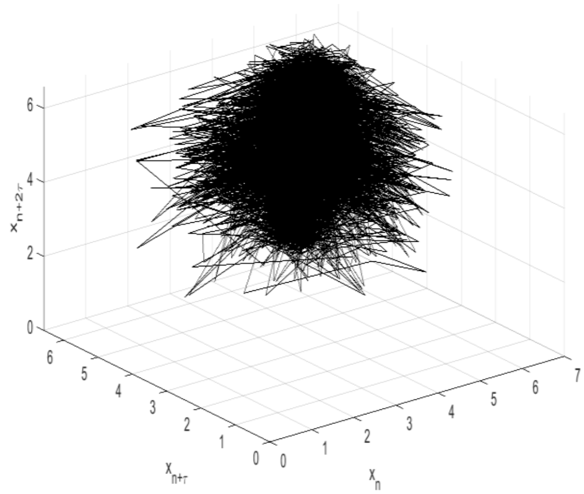
(c) Jos



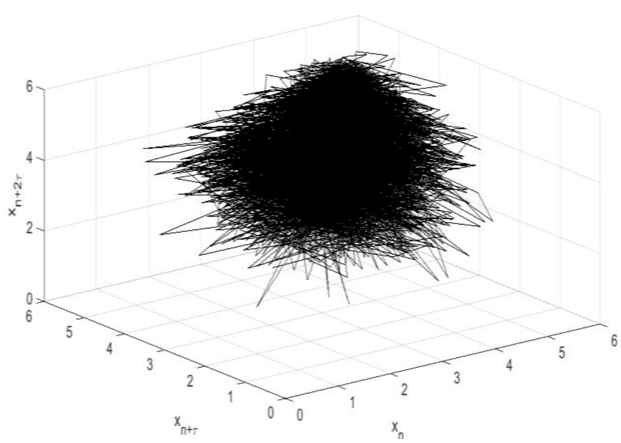
(d) Minna



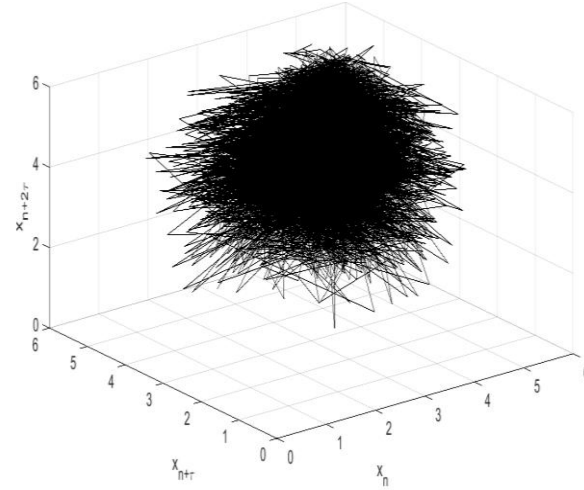
(e) Sokoto



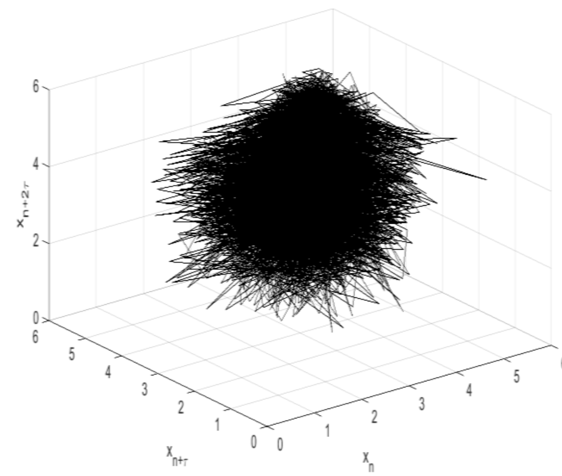
(f) Katsina



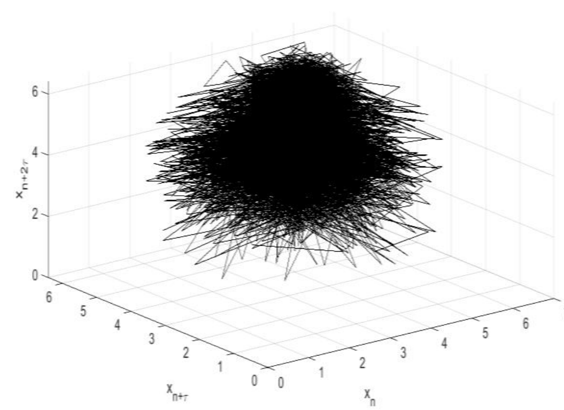
(g) Kaduna



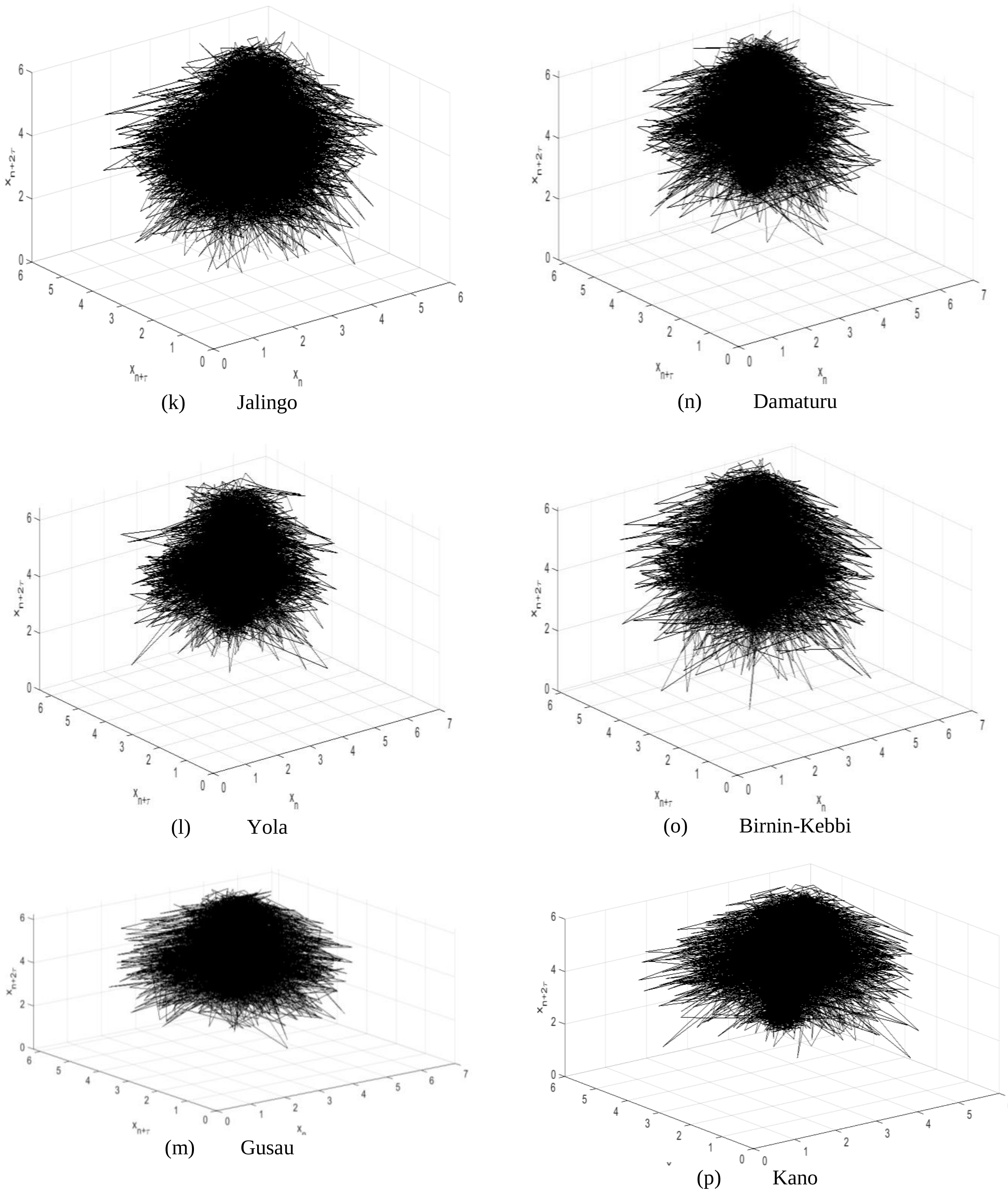
(h) Bauchi



(i) FCT Abuja



(j) Maiduguri



(k) Jalingo (n) Damaturu

(l) Yola (o) Birnin-Kebbi

(m) Gusau (p) Kano

Figure 2: Phase portrait of ETo across Northern Nigeria

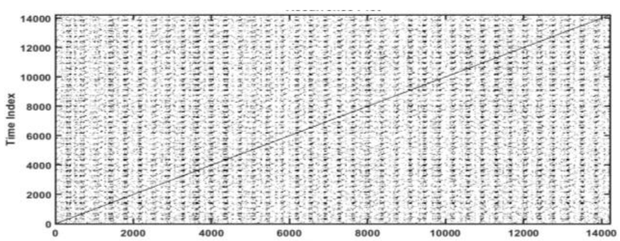
The results of the 3-dimensional phase portraits of the 16 northern states in Nigeria from 1984-2022 showed a spongy bird's nest-like structures showing a distinct line indicating chaotic/stochastic (random) behavior in all the locations [7]. The phase portraits result from this study agrees with that [15] where they also reported a chaotic behavior. The reason for this chaotic behavior might be due to the random behavior in the recorded weather data across the 16 locations across Northern Nigeria.

Recurrence plots of reference evapotranspiration

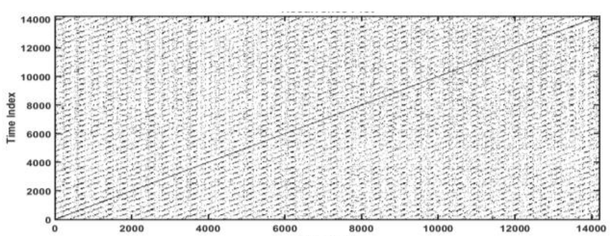
Recurrence Plot is a plot showing for each moment in time an ETo value return at another time in a dynamical system. The first time of the initial ETo is plotted against the recurrence time it reappears in a phase plane. Therefore, it presents a binary plot time series for the purpose of visualizing the patterns of ETo recurrence in each of the 16 Northern states in Nigeria as presented in Fig. 3. Recurrence plot is computed using 2 by 2-time matrix to derive a Heaviside function consisting of 0 and 1. The Heaviside function is then multiplied with the recurrence threshold limit, ε to derive a recurrence plot value.The recurrence threshold limit, ε is estimated using the expression:

$$\mathcal{E} = \sqrt{m \times 10\% \text{ of the fluctuations of the signal (data)}}$$

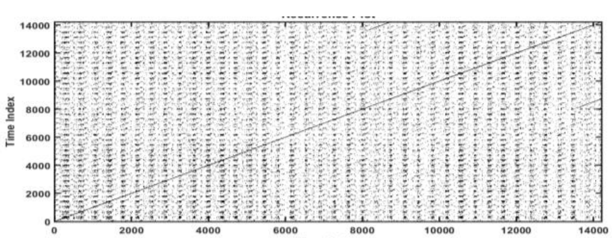
Where m is the embedding dimension and the fluctuation of the signal is a specific fraction of its phase space diameter or its standard deviation, δ [3].



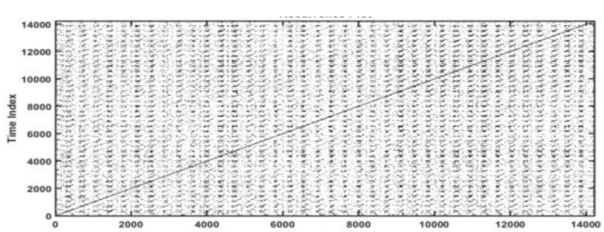
(a) Makurdi, $\varepsilon = 0.245$



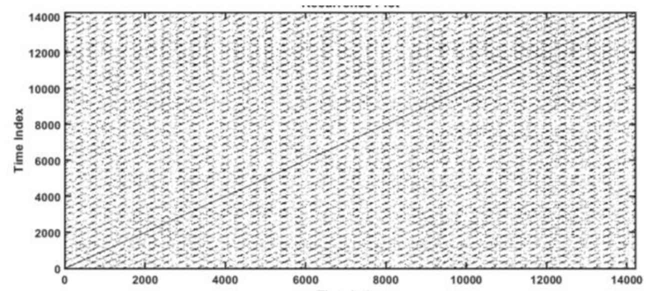
(b) Dutse, $\varepsilon = 0.25$



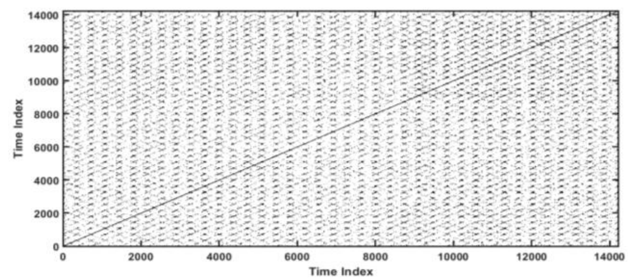
(c) Jos, $\varepsilon = 0.24$



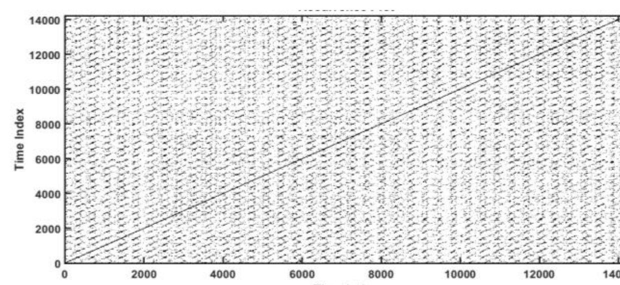
(d) Minna, $\varepsilon = 0.245$



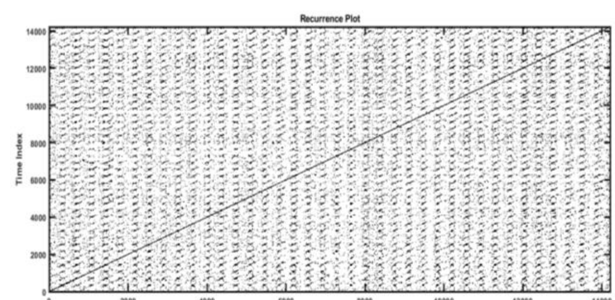
(e) Sokoto, $\varepsilon = 0.24$



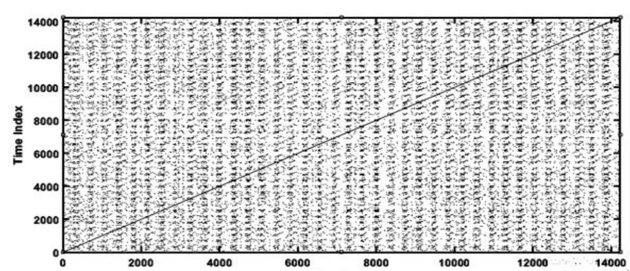
(f) Katsina, $\varepsilon = 0.20$



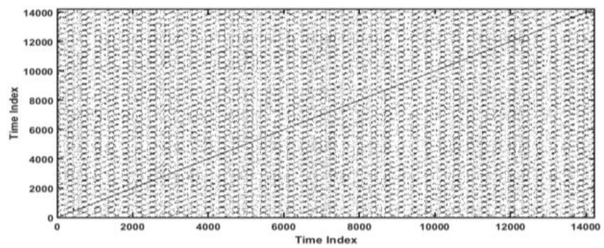
(g) Kaduna, $\varepsilon = 0.24$



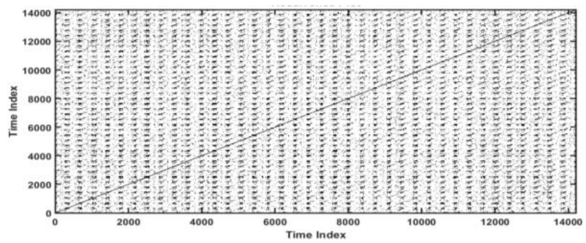
(h) Bauchi, $\varepsilon = 0.24$



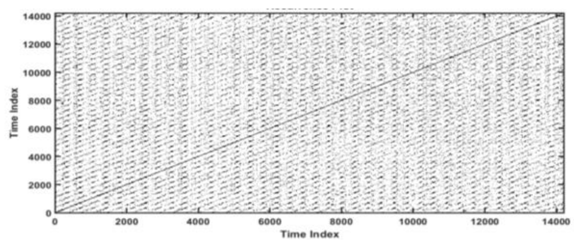
(i) FCT Abuja, $\varepsilon = 0.21$



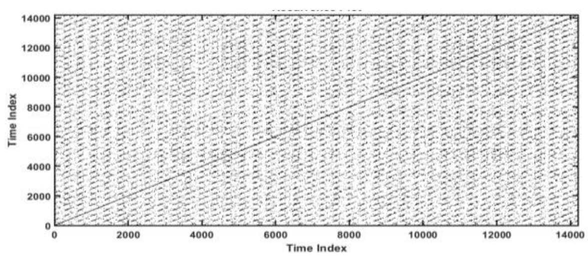
(j) Maiduguri, $\varepsilon = 0.28$



(k) Jalingo, $\varepsilon = 0.25$

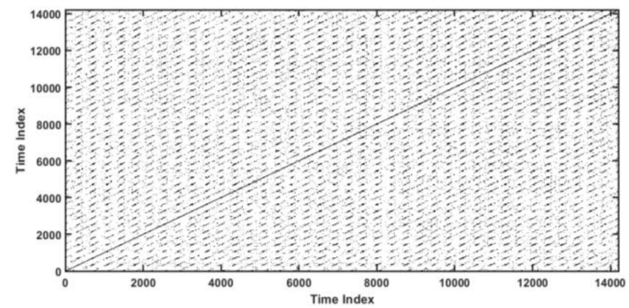


(l) Yola ,$\varepsilon = 0.25$

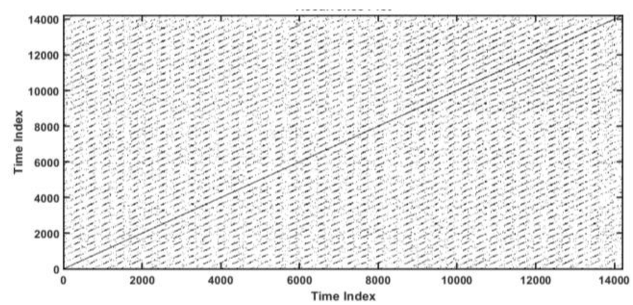


(m) Gusau, $\varepsilon = 0.25$

(n) Damaturu, $\varepsilon = 0.24$



(o) Birnin-Kebbi, $\varepsilon = 0.20$



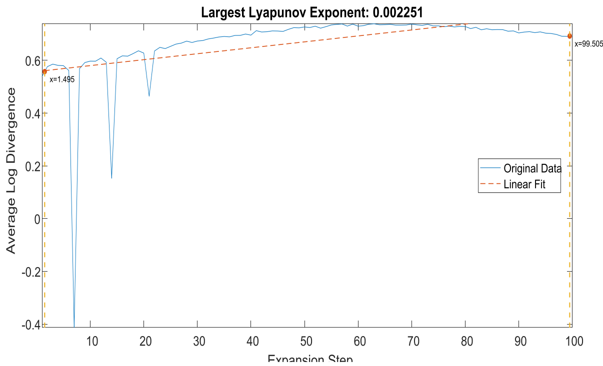
(p) Kano, $\varepsilon = 0.20$

Figure 3: Recurrence plot of ETo across northern Nigeria

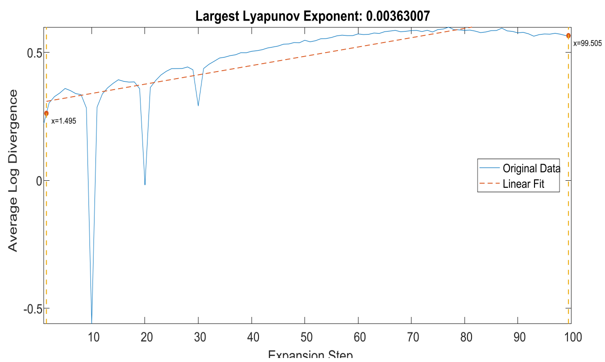
The results of the RP displayed in Fig. 3 showed points that are arranged in shorter diagonal lines. The short correlated diagonal lines indicated that the dynamical system of ETo is chaotic which is deterministic in nature across Northern Nigeria. In addition, the presence of dark short diagonals and white ribbons is an indication of seasonality in the daily ETo across the North Nigeria. This implies that short-term predictability of the daily ETo is possible. Also, it is observed that short RP lines are more correlated in the North West (Sokoto, Katsina, Gusau, etc.) than in the North Central (Makurdi, Abuja, Jos) and North East (Maiduguri, Yola and Jalingo). This implies more randomness in the ETo in North central and North East than in the North West. This is attributed to more human anthropogenic activities in the region leading to greenhouse effect and global warming which causes more fluctuations in daily ETo [3].

Lyapunov exponent for reference evapotranspiration

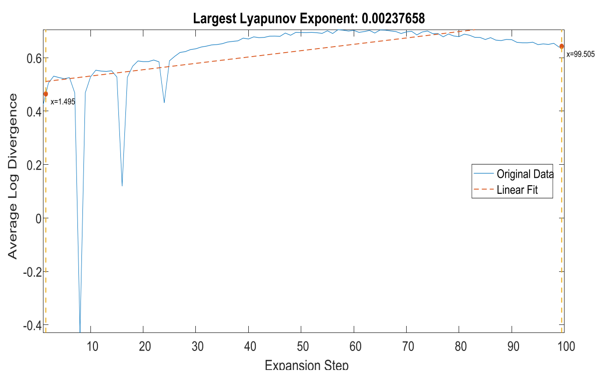
Lyapunov Exponent is a plot that defines the divergence of two closest ETo. The plot consists of average log divergence of two closest ETo and the expansion step which is the mean time difference between them. The log of the function of the difference between two closest ETo is first computed, then the mean time difference between them is computed. Based on this, a two-dimensional plot is initiated using the log divergence of two closest ETo against the expansion step. From the plot the lowest apex point and the highest apex point based on the random fluctuation of the plot is link linearly to derive a slope of the entire plot. The linear slope of this plot is defined as the Largest Lyapunov Exponent (λ) of each state. The results of λ of the across the 16 Northern states of Nigeria from 1984 – 2022 is presented in Fig. 4.



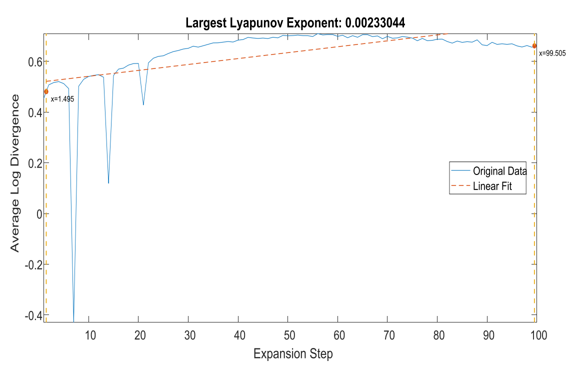
(a) Makurdi; $\lambda_1 = 0.002251$



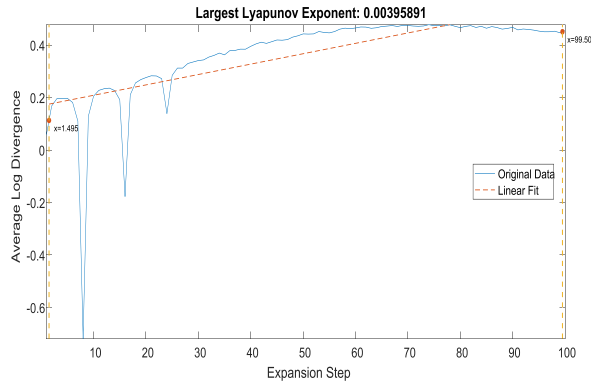
(b) Dutse; $\lambda_1 = 0.003630$



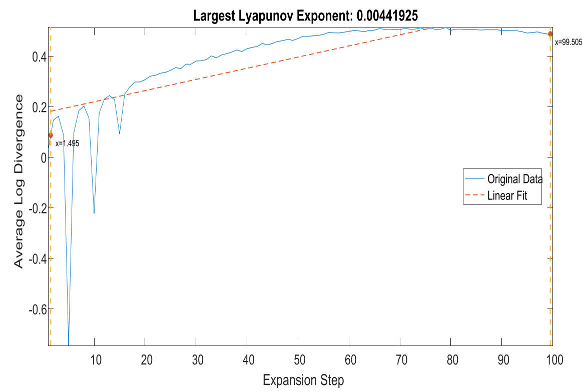
(c) Jos; $\lambda_1 = 0.002377$



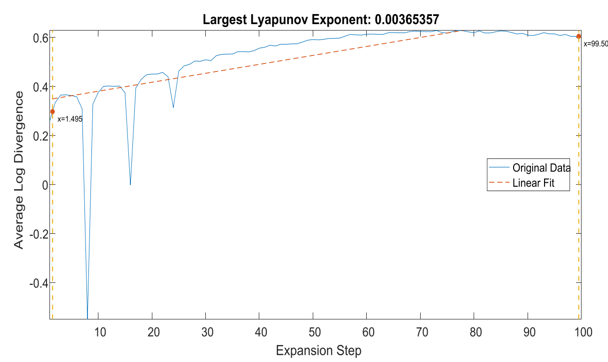
(d) Minna; $\lambda_1 = 0.002330$



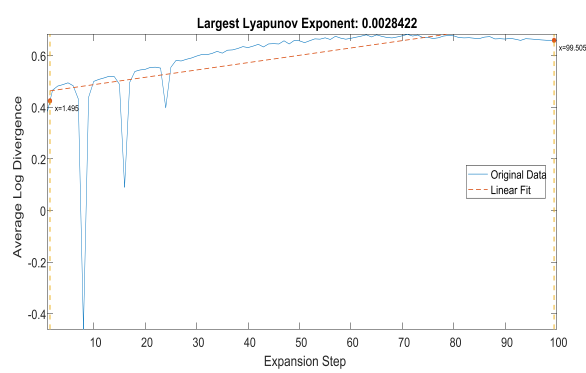
(e) Sokoto; $\lambda_1 = 0.003959$



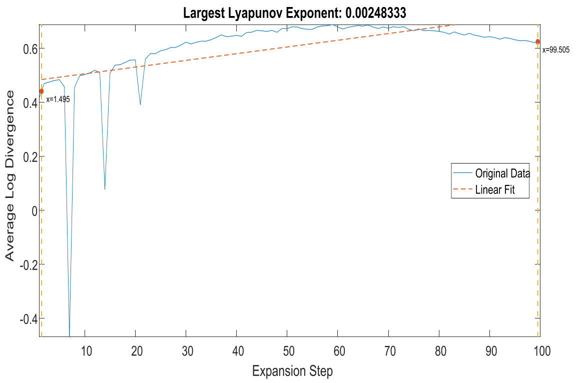
(f) Katsina; $\lambda_1 = 0.004419$



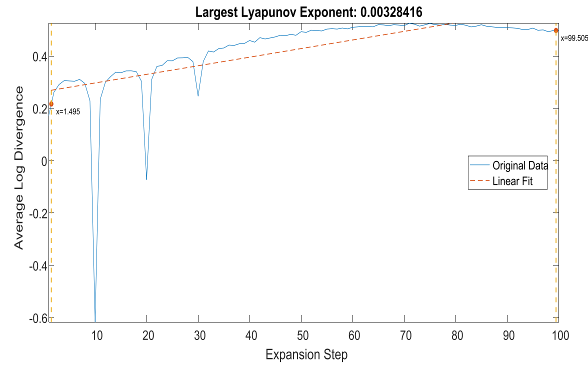
(g) Kaduna; $\lambda_1 = 0.003654$



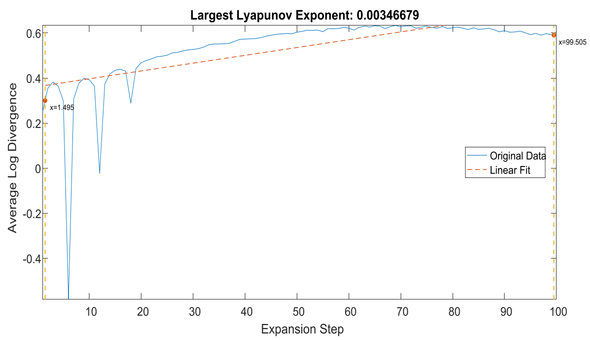
(h) Bauchi; $\lambda_1 = 0.003630$



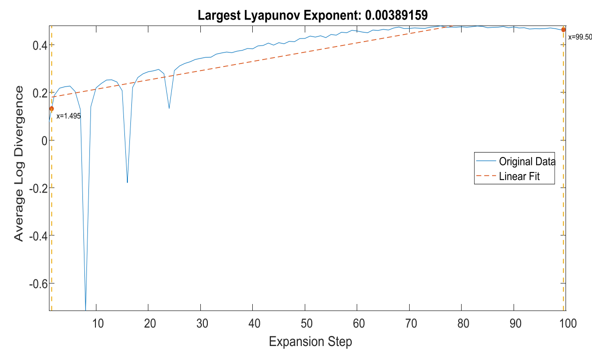
(i) FCT Abuja; $\lambda_1 = 0.002483$



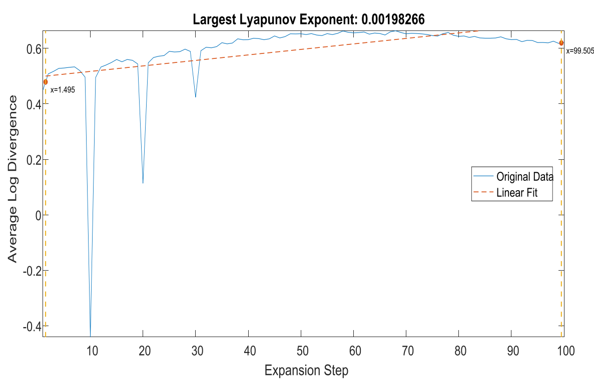
(m) Gusau; $\lambda_1 = 0.003284$



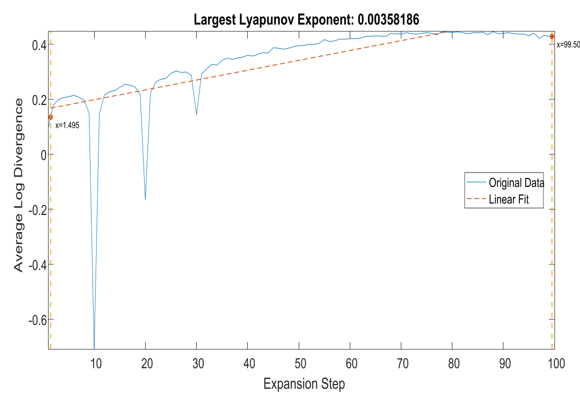
(j) Maiduguri; $\lambda_1 = 0.003467$



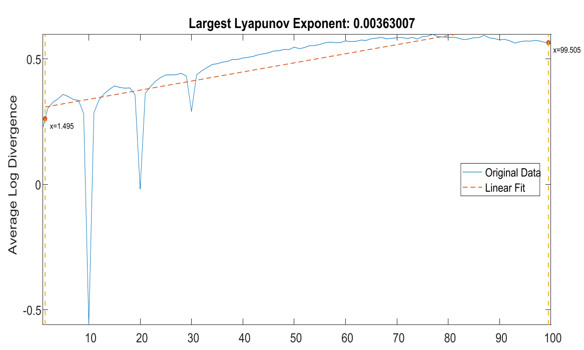
(n) Damaturu; $\lambda_1 = 0.003892$



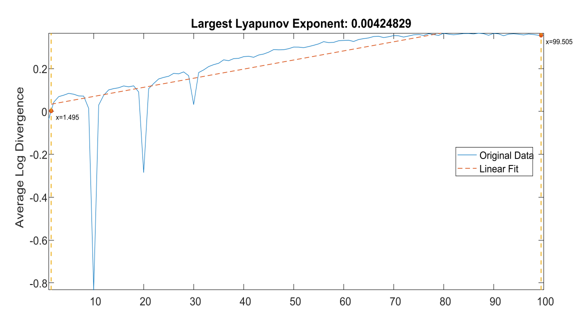
(k) Jalingo; $\lambda_1 = 0.001983$



(o) Birnin-Kebbi; $\lambda_1 = 0.003582$



(l) Yola; $\lambda_1 = 0.003630$



(p) Kano; $\lambda_1 = 0.003582$

Figure 4: Lyapunov exponent of ETo across Northern Nigeria

The largest Lyapunov Exponents for the mean daily ETo in all the locations were computed and were all observed to be positive. This is an indication of deterministic chaos in the daily ETo. The plot also indicated aperiodic behavior that is sensitive to initial conditions and thus exhibits topological mixing for dense states. From the plot, it was observed that the Lyapunov exponent of daily ETo increases significantly from south to North i.e. from Makurdi in the North Central (λ= 0.002251/day) to Katsina in the North West (λ= 0.004419/day), while longitudinally, an insignificant decrease was recorded with a value of λ= 0.003582/day, in Birnin-Kebbi to λ= 0.003467/day in Maiduguri. Overall, Katsina recorded the highest degree of chaos in ETo as the highest value of Lyapunov exponent of 0.004419 was computed while Jalingo recorded the lowest degree of chaos (λ = 0.001983). The result from this study agrees with that of many researchers [13, 15, 17 and 18] where they also reported a chaotic behavior which was because of strong seasonality.

Conclusion

Based on the findings of this study, it is concluded that the dynamical system of reference evapotranspiration in sixteen locations across northern Nigeria is unstable and chaotic. This finding was based on the sponge-like shape of phase portraits plot of reference evapotranspiration. Also, the Recurrence plot in all the 16 locations showed a presence of dark short diagonals and white ribbons which is an indication of seasonality in the daily ETo across the North Nigeria. This implies that short-term predictability of the daily ETo is possible. Also, it is observed that short Recurrence plot lines are more correlated in the North-West (Sokoto, Katsina, Gusau etc.) than in the North-Central (Makurdi, Abuja, Jos) and North-East (Maiduguri, Yola and Jalingo). This implies more randomness in the ETo in North-Central and North-East than in the North-West. This is attributed to more human anthropogenic activities in the region leading to greenhouse effect and global warming which causes more fluctuations in daily ETo.

The plot of Lyapunov exponent showed that the daily ETo increases significantly from south to North i.e. from Makurdi in the North Central (λ= 0.002251/day) to Katsina in the North West (λ= 0.004419/day), while longitudinally, an insignificant decrease was recorded with a value of λ= 0.003582/day, in Birnin-Kebbi to λ= 0.003467/day in Maiduguri. Overall, Katsina recorded the highest degree of chaos in ETo as the highest value of Lyapunov exponent of 0.004419 was computed while Jalingo recorded the lowest degree of chaos (λ = 0.001983).

Conflict of interest: The authors declare no conflict of interest whatsoever.

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