

DISTANCE-BASED TOPOLOGICAL INDICES OF THE MOLECULAR STRUCTURE OF SOME DRUGS FOR TREATING CANCER PATIENTS

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ABSTRACT

Topological descriptors are key tools for analyzing the intrinsic properties of chemical graphs and nanostructures, capturing molecular symmetry through graph invariants such as vertex degree, distance, and spectrum. This study investigates vertex-distance-based indices, namely: the Wiener and Hosoya indices, for selected anticancer drugs. Using linear regression models, the indices were correlated with the drugs' physicochemical and biological properties through a Quantitative Structure–Property Relationship (QSPR) analysis, providing insight into their linear relationships and potential applications in drug design.

Keywords: Connected Graph, Wiener Index, Hosoya Polynomial, Molecular Structure, Linear Regression

INTRODUCTION

A graph G can be considered as an ordered triple (V, E, ρ) , where $V = \{v_1, v_2, v_3, \dots\}$ is called the vertex set of G and the elements of V are called the vertices (or points or nodes); $E = \{e_1, e_2, e_3, \dots\}$ is called the edge set of G and the elements of E are called edges (or lines or arcs); and ρ is called the adjacency relation, defined by $\rho : E \rightarrow V \times V$, which defines the association between each edge with the vertex pairs of G (Biggs, 1933). The distance between two vertices u and v in a graph G , denoted by $d(u, v)$, is the length (number of edges) of a shortest path (also called a graph geodesic) connecting them. This distance is also known as the geodesic distance. Topological index is a numeric quantity that tells us about the whole structure of a graph (Gao *et al.*, 2016). Despite significant advances in cancer drug discovery, the process of developing new and effective anticancer agents remains costly, time-consuming, and experimentally intensive, (Shanmukha *et al.*, 2020). Also, predicting the physical, chemical, biological, and pharmacological properties of cancer drugs from their molecular structures remains a significant challenge in pharmaceutical research. Traditional experimental methods for evaluating the physicochemical and biological properties of drug candidates are often limited by resource constraints and the complexity of molecular interactions. As a result, there is a critical need for efficient, reliable, and predictive computational tools that can accelerate drug design and optimization (Ravi & Desikan, 2023).

Topological indices are numerical invariant derived from molecular graphs that quantitatively represent structural features including connectivity, branching, and molecular complexity. Quantitative Structure–Property Relationship (QSPR) models using topological indices have demonstrated significant predictive capabilities. These models establish correlations

between topological indices and important physicochemical properties including: Boiling point (BP), Flash point (FP), Molar volume (MV), Molecular weight, Complexity, Molar refractivity, Polarizability, and Surface area (Prana *et al.*, 2017; Sjanmukha *et al.*, 2022; Shi *et al.*, 2025).

The Wiener index $W(G)$ of a graph G was the first topological index to be used in Chemistry. It was introduced as the path number of G by Wiener in 1947, whereby he carried out some investigations on the relationship between the structures and properties of saturated hydrocarbons (Wiener, 1947). The index was viewed as the sum of all shortest carbon-carbon bond paths in a hydrocarbon. This simple numerical representation of a molecule has shown to be a very useful quantity in the study of quantitative structure–property relationships (QSPR). The definition of the Wiener index in terms of distances between vertices of a graph was first given by Hosoya in 1971.

In this era of rapid technological development, chemical and pharmaceutical techniques in recent years have been rapidly evolving, and thus a large number of new nanomaterials, crystalline materials, and drugs emerge yearly. To investigate the physio-chemical properties of such large number of new compounds and new drugs require a large amount of chemical experiments, thereby increasing the workload, resources and time-frame of the pharmaceutical and chemical researchers. Fortunately, chemical based experiments showed that there was a strong connection between topology molecular structures and their physical behaviors, chemical characteristics, and biological features such as flash point, boiling point, melting point, and toxicity of drugs (Prana *et al.*, 2017). The topological index of a molecular structure is a nonempirical numerical quantity which quantities are its molecular structure and branching pattern (Nandini *et al.*, 2021).

Many studies in this area have since then published. The study on the family of smart polymer which is widely used in anticancer manufacturing was conducted (Wei *et al.*, 2021). Investigations of various degree-based, neighborhood-based and eccentricity-based topological indices by considering edge partition method for the molecular structure of anti-cancer drug, Pectin, without going to the wet lab (Puneeth *et al.*, 2024). The general Randic index, atom bond connectivity, geometric arithmetic index, general harmonic index, Zagreb indices and the 5th version of atom bond connectivity index and 4th version of the geometric arithmetic index, for the molecular graph of Pectin was computed (Kopperundevi, 2025; Xiaolong *et al.*, 2025 & Sudhakar, 2009). Also, various topological indices on some anticancer drugs was defined in order to help researchers with the knowledge of the physical characteristics and chemical reaction associated with them (Zaman *et al.*, 2024; Wazzan & Ozalan, 2023; Arokiaraj *et al.*, 2023, Arokiaraj *et al.*, 2025).

Specifically, this study investigates relationship between some structural information of the properties of some anti-cancer drugs and the Wiener Index of their molecular graphs, with the aim of formulating a QSPR mathematical model to predict the physiochemical properties of these drugs.

MATERIALS AND METHODS

Wiener Index of a Connected Graph

The Wiener index of a connected graph G is defined as Prana *et al.* (2017):

$$W(G) = \frac{1}{2} \sum_{(u,v) \in E(G)} d(u,v) \quad (1)$$

where $d(u, v)$ is the distance between the vertex u and the vertex v , and $E(G)$ is the edge in G .

Some molecular structures of the anti-cancer drugs are provided in Figures 1 – 10.

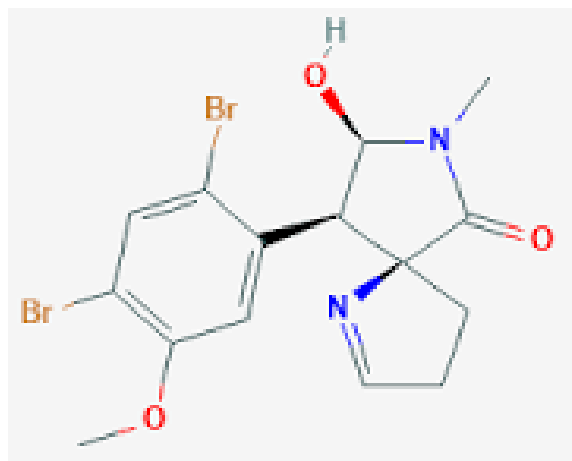


Figure 1: Amathaspiramide E.

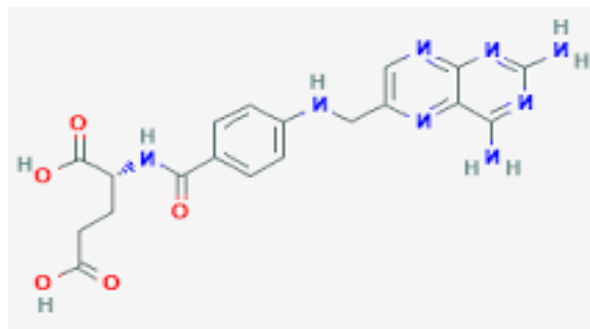


Figure 2: Aminopterin

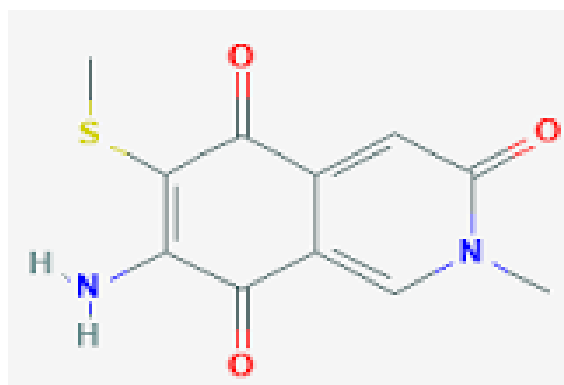


Figure 3: Perfragilin A.

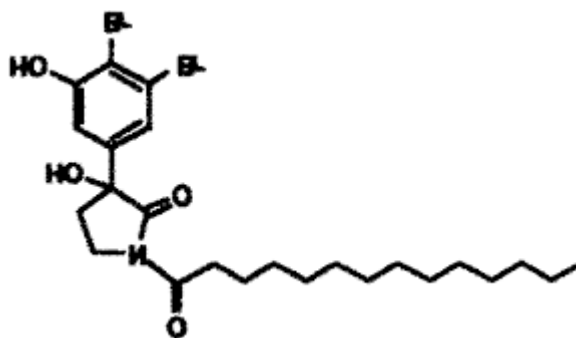


Figure 4: Convolutamide A.

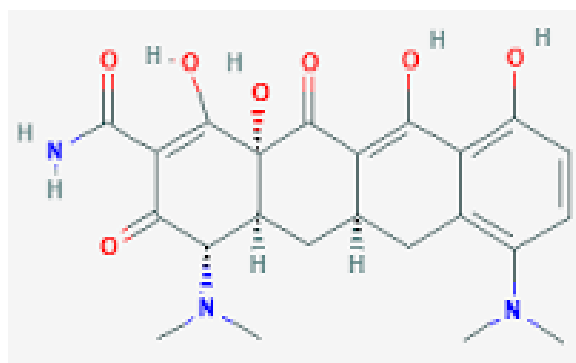


Figure 5: Minocycline

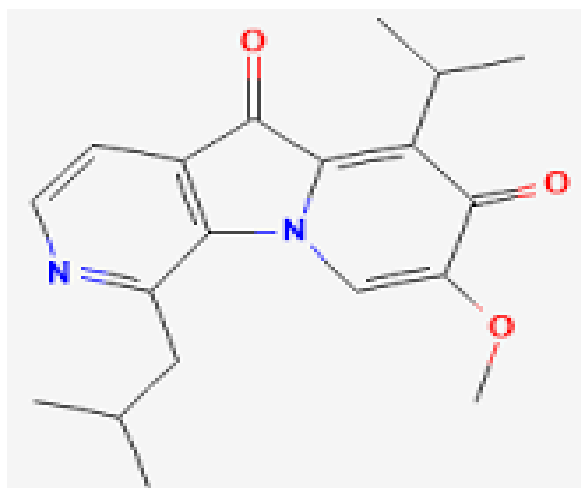


Figure 6: Pterocellin B.



Figure 9: Convulutamine F.

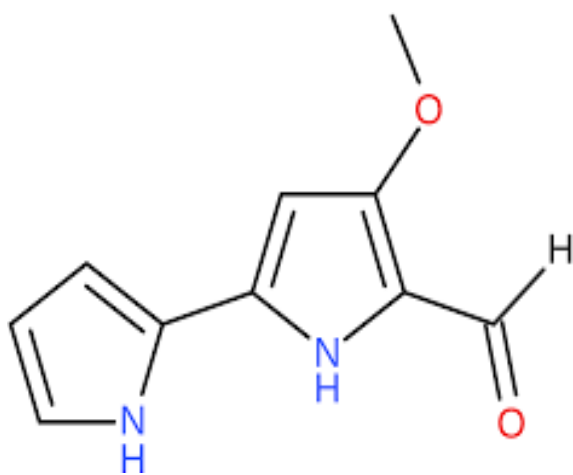


Figure 7: Tambjamine K.

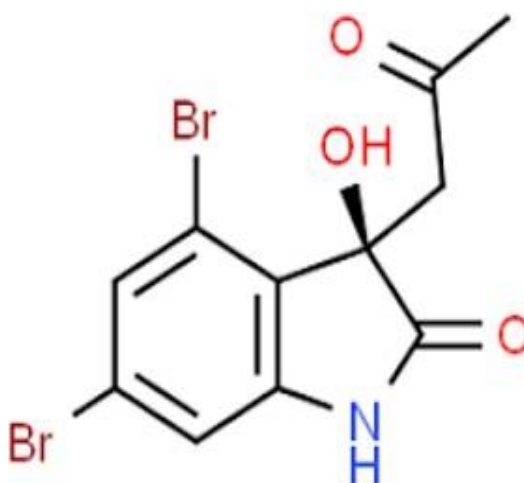


Fig 10: Convolutamydine A.

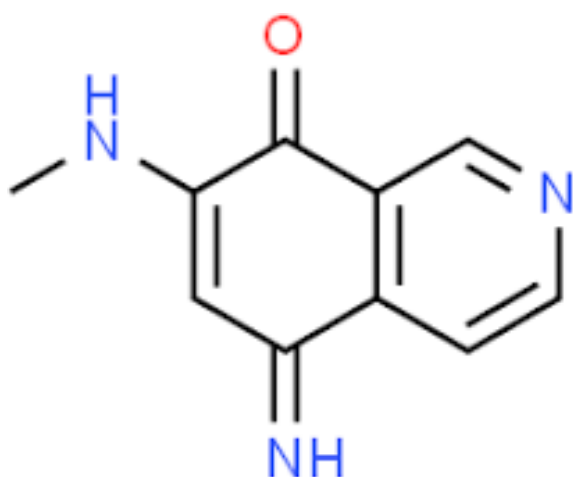


Figure 8: Caulibugulone E.

Hosoya Polynomial of a Connected Graph

The Hosoya polynomial of a connected graph G is defined to be,

$$H(G, x) = \frac{1}{2} \sum_{(u,v) \in E(G)} x^{d(u,v)} \quad (2)$$

Linear Regression Model

The regression model is defined as;

$$P = A + B(TI) \quad (3)$$

where P is the physical property of the drug, A is a constant and B is the regression coefficient and TI represent the Topological Indices of the molecular graph of these drugs.

RESULTS AND DISCUSSION

Computation of the Wiener Indices of the Molecular Graphs

Molecular graph is a graph obtained from a chemical structure where its vertices represent the atoms and the bond connecting them at the edges. The molecular graphs of the compounds are given in Figures 11 – 21.

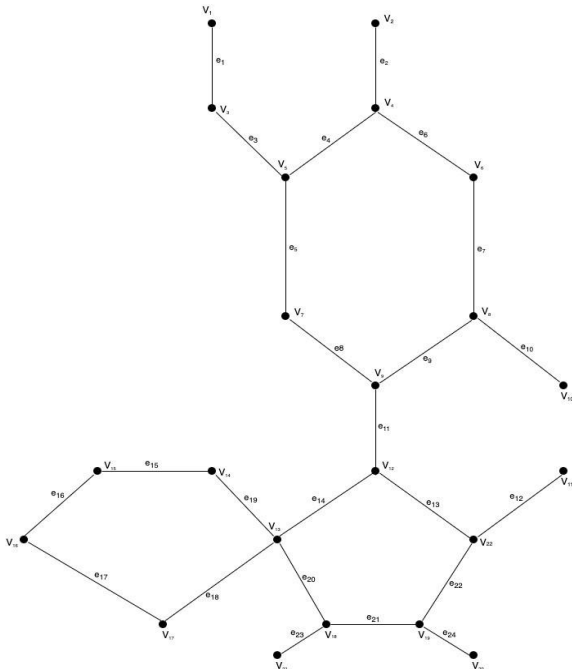


Figure 11: Molecular graph of amathaspiramide E drugs

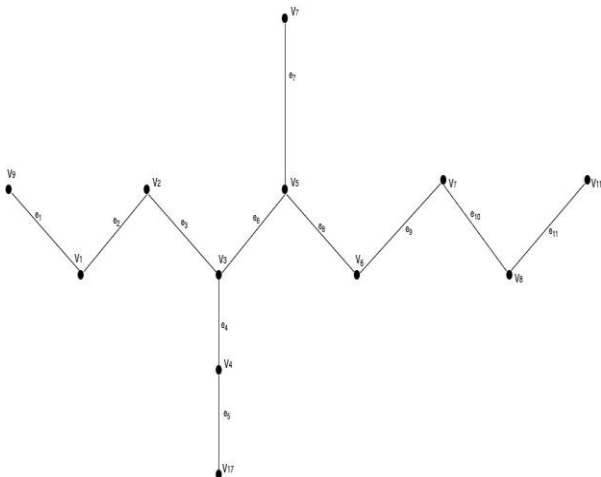


Figure 12: Molecular graph carmustine drugs

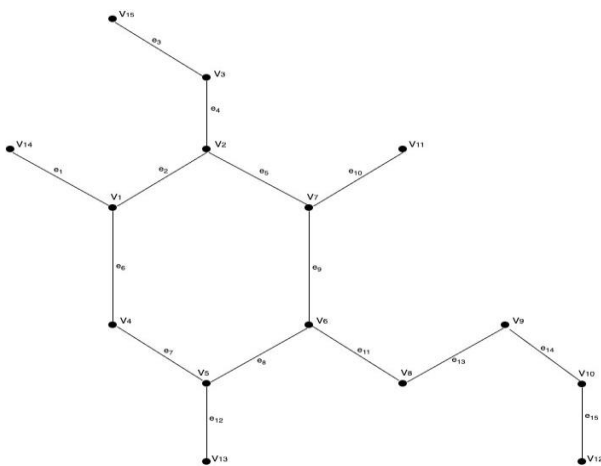


Figure 13: Molecular graph convolutamine F drugs

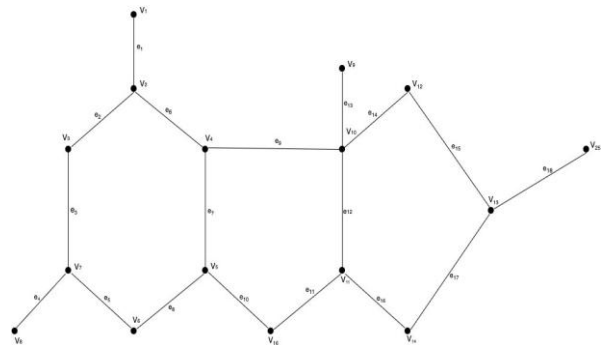


Figure 14: Molecular graph convolutamy A drugs

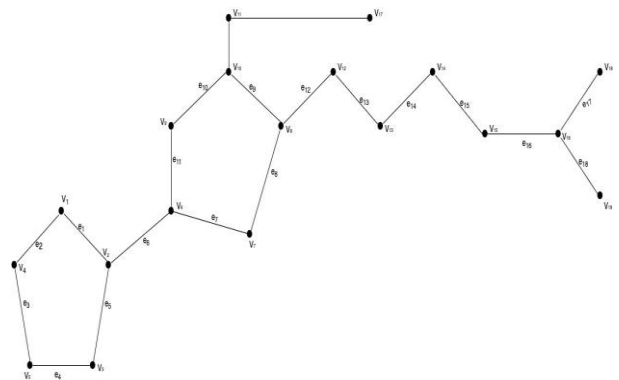


Figure 15: Molecular graph tambjamine K drugs

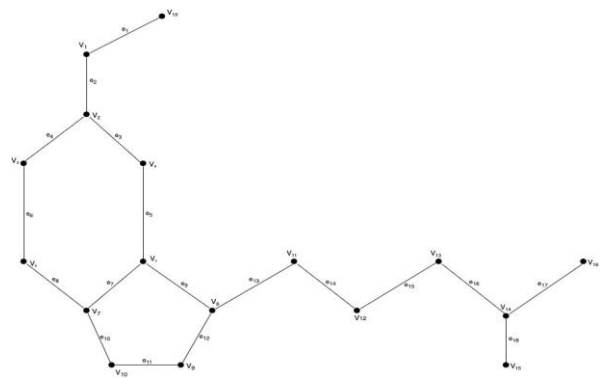


Figure 16: Molecular graph of melatonin drugs

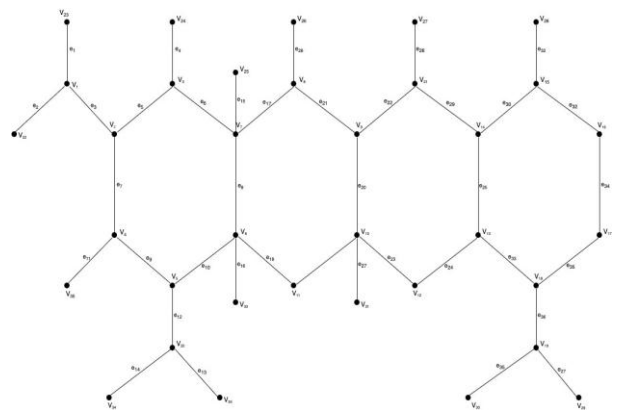


Figure 17: Molecular graph of minocycline drugs

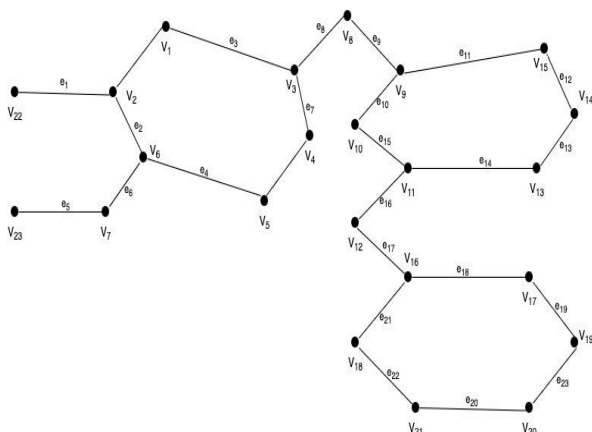


Figure 18: Molecular graph of pterocellin B drugs

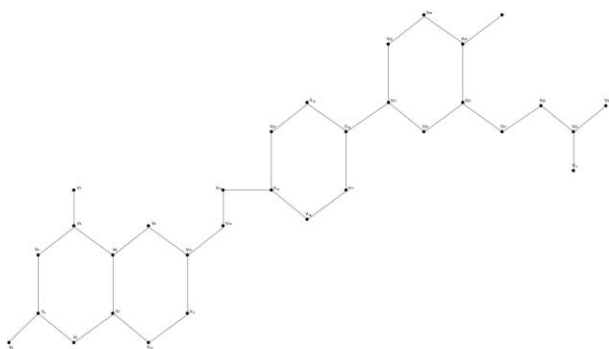


Figure 19: Molecular graph of aminopterin drugs

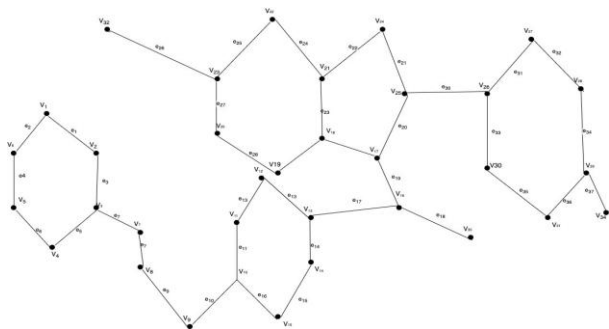


Figure 20: Molecular graph of theaflavin drugs

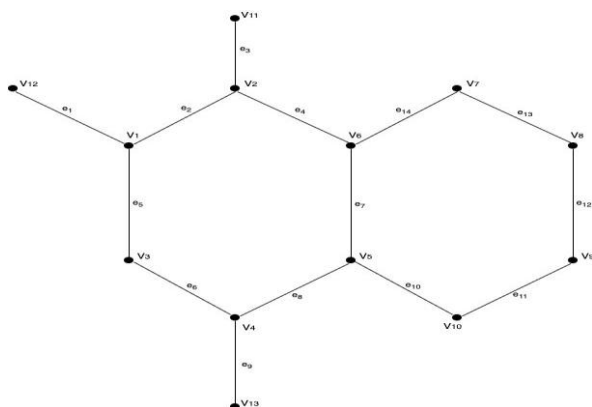


Figure 21: Molecular graph caulibugulone E drugs

Using Equation (1), we have the following computation of the Wiener index of the molecular graphs in Figures 11 -21;

Table 1: Wiener indices for some molecular graphs

S/N	Drugs	Disease	Index
1	Theaflavin	Cancer	2622
2	Convolutamide A	Cancer	1573.5
3	Tambjamine K	Cancer	400
4	Amathaspiramide E	Cancer	456.5
5	Pterocellin B	Cancer	676
5	Minocycline	Cancer	1293.5
6	Carmustine	Cancer	113
7	Caulibugulone E	Cancer	108.5
8	Convultamy A	Cancer	191
9	Aminopterin	Cancer	1657
10	Melatonin	Cancer	280.5
11	Convolutamine F	Cancer	174.5

Computation of the Hosoya Polynomial of the Molecular graphs

Let M_p denote the molecular graph of the Mepacrine chemical compound. Using Equation 2, its Hosoya polynomial is computed as: $H(M_p, x) = \frac{1}{2}(x^{191} + x^{164} + x^{142} + x^{148} + x^{124} + x^{101} + x^{113} + x^{116} + x^{96} + x^{105} + x^{113} + x^{124} + x^{108} + x^{93} + x^{79} + x^{66} + x^{54} + x^{43} + x^{33} + x^{40} + x^{24} + x^{17} + x^{11} + x^9 + x^7 + x^6 + x)$

Let C_m denote the molecular graph of the Camostat chemical compound. Using Equation 2, its Hosoya polynomial is computed as:

$$H(C_m, x) = \frac{1}{2}(x^{282} + x^{256} + x^{279} + x^{227} + x^{202} + x^{182} + x^{161} + x^{177} + x^{156} + x^{136} + x^{117} + x^{133} + x^{99} + x^{83} + x^{60} + x^{67} + x^{55} + x^{42} + x^{34} + x^{25} + x^{31} + x^{17} + x^{11} + x^6 + x^8 + x^2 + x^2)$$

Let C_A denote the molecular graph of the Convolutamide A chemical compound. Using Equation 2, its Hosoya polynomial is computed as:

$$H(C_A, x) = \frac{1}{2}(x^{208} + x^{227} + x^{205} + x^{184} + x^{164} + x^{181} + x^{138} + x^{122} + x^{129} + x^{138} + x^{142} + x^{108} + x^{102} + x^{97} + x^{93} + x^{90} + x^{88} + x^{87} + x^{87} + x^{88} + x^{90} + x^{93} + x^{97} + x^{102} + x^{34} + x^{22} + x^{16} + x^{12} + x^3)$$

Let P_A denote the molecular graph of the Perfragilin A chemical compound. Using Equation 2, its Hosoya polynomial is computed as:

$$H(P_A, x) = \frac{1}{2}(x^{41} + x^{43} + x^{41} + x^{30} + x^{28} + x^{26} + x^{26} + x^{26} + x^{24} + x^{21} + x^{23} + x^{22} + x^{18} + x^7 + x^3)$$

In the same manner, others Hosoya polynomials are obtained.

Linear Regression Analysis and QSPR Model of the Drugs

Using equation (3) and R software, we have the following Table;

Table 2: Statistical parameters for the linear QSPR model for TI of anticancer drugs

Physical Properties	N	A	b	r	F	P	Indicator
Boiling Point	15	146.360	0.085	0.812	21.320	0.001	Significant
Melting Point	13	384.414	0.213	0.884	46.390	0.000	Significant
Enthalpy	14	64.847	0.027	0.858	33.493	0.000	Significant
Molar Refraction	15	56.901	0.043	0.964	170.634	0.000	Significant

The QSPR Model for the properties of anticancer molecular drugs is:

$$BP = 143.360 + 0.085(W(G)) \quad (4)$$

$$MP = 384.414 + 0.213W(G) \quad (5)$$

$$E = 530.267 + 0.027W(G) \quad (6)$$

$$MR = 59.901 + 0.043W(G) \quad (7)$$

Where $W(G)$ denote the Wiener index of the molecular graph G , BP , MP , E , and MR represents the Boiling Point, Melting Point, Enthalpy, and Molar Refraction, respectively.

Table 3: Discussion table for obtained results vs literature (cancer drugs)

Physical Properties	r	r in the Literature	Source	Remarks
Boiling Point	0.812	0.74-0.96	Kopperundevi (2025)	Solid linear fit
Melting Point	0.884	0.56-0.90	Meharban <i>et al.</i> (2024)	On high side
Enthalpy	0.858	0.73-0.96	Kopperundevi (2025)	Comparable
Molar Refraction	0.964	0.88-0.99	Xiaolong <i>et al.</i> (2025)	Very strong, Typical best

The reports from Table (4.2), the fit of simple linear models of Equation (3.3) for each physicochemical property (P) (BP, MP, Enthalpy, MR) versus the Wiener topological index defined in Equation (3.1). The Sample size (N) is modest (13–15), typical for cancer-drug QSPR sets. It can be seen from Table (4.2) that all the correlation coefficients, $r \geq 0.8$, so the linear TI–property relationships are strong, especially for molar refraction, matching patterns seen in cervical and breast-cancer drug studies where MR often shows the tightest TI correlation. The value of F ranges from 21 to 171, all $p \leq 0.001$, meaning each regression is statistically significant at conventional levels, similar to reported F and p for degree-based and temperature-based TIs in cancer-drug QSPR (Qin *et al.*, 2025).

The values of r and p values fall within the “good QSPR” band used as acceptance criteria in cervical, lung, breast, kidney and general anticancer drug studies, where $r > 0.7$ and $p < 0.05$ define acceptable models in the work of Meharban *et al.* (2024) and Xiaolong *et al.* (2025). The especially high correlation for molar refraction (0.964) mirrors multiple works where MR is the best-predicted property across cancer drug sets, as seen in the case of Meharban *et al.* (2024).

Boiling Point: (BP)

The intercept is 143.360: predicted BP when $W(G)=0$ (a theoretical baseline) and the slope is 0.085, meaning, for each unit increase in $W(G)$, BP increases by 0.085 °C. This means that more complex and connected molecules higher than $W(G)$ are predicted to boil at higher temperatures, consistent with degree- and coindex-based QSPR on cancer drugs.

Melting point (MP):

The intercept is 384.414, which is a baseline melting point, while the slope is 0.213, meaning, each 1-unit increase in $W(G)$ raises MP by 0.213 °C. This means that the structural growth captured by $W(G)$ tends to

stabilize the solid phase, raising melting point, as seen in other anticancer QSPR models.

Enthalpy of vaporization (E):

The base enthalpy value is 530.267, which is the intercept. The slope is 0.027 in which each unit increase in $W(G)$ raises E by 0.027 kJ/mol. This shows that the larger/more connected molecules require more energy to vaporize, matching trends in lung, colorectal and general cancer-drug QSPR.

Molar refraction (MR)

The base refractivity at $W(G)=0$ has the intercept, 59.901 and slope, 0.043, which shows that each unit increase in $W(G)$ raises MR by 0.043 cm³ (typical MR unit). This means that, MR, closely related to volume and polarizability, increases strongly with $W(G)$.

CONCLUSION

The Wiener indices of the various drugs have been computed with the Hosoya polynomials determined as shown in Table 1. From Tables 2 and 3, the regression model value r is more than 0.6 and p value show less than 0.05. Hence, it can be concluded that all the physicochemical properties are highly significant. The QSPR Models shows that there is a linear relationship between the physicochemical properties of these drugs and its topological indices. These models can be used by researchers to predicts future properties of such drugs. We also observed that the first derivative of their Hosoya polynomials at the point 1, will give us their Wiener Indices. This validates the accuracy of our results. The results as shown in the QSPR Models are important to predict or guess the properties of such drugs and study the topology of other structure of molecules, and can also be used in drug design.

Conflict of interest: The authors declare no conflict of interest.

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