TERTIARY EDUCATION TRUST FUND

PHYSICAL SCIENCES

ON PERFORMANCE OF SOME METHODS OF DETECTING NONLINEARITY IN STATIONARY AND NON-STATIONARY TIME SERIES DATA

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Date Manuscript Received: 12/07/2016 Accepted: 23/12/2016 Published: December, 2016

ABSTRACT
There has been growing interest in exploiting potential forecast gains from the nonlinear structure of autoregressive time series. Several models are available to fit nonlinear time series data. However, before investigating specific nonlinear models for time series data,it is desirable to have a test of nonlinearity in the data. And since most of real life data collected are non-stationary, there is need to investigate which of these test is suitable for stationary and non-stationary data. Statistical tests have been proposed in the literature to help analysts to check for the presence of nonlinearities in observed time series, these tests include Keenan and Tsay tests, and they have been used under the assumption that data is stationary. However, in this paper, we investigated the performance of these two tests for the stationary and non-stationary data. The effect of the stationarity and non-stationarity were studied on simulated data based on general class of linear and nonlinear autoregressive structures using R-software. The powers of tests were compared at different sample sizes for the two cases. It was observed that the Tsay F-test performed better than Keenan's tests with little order of autoregressive and increase in sample size when data is non-stationary and vice-versa when data is stationary. Finally, we provided illustrative examples by applying the statistical tests to real life datasets and results obtained were desirable.

Key words: *Nonlinearity, Tsay's F Test, Keenan's Test, Stationarity and Non-Stationarity*

INTRODUCTION

Several techniques used in time series modeling assume linear relationships among variables. However, in some cases, variations in data do not show simple linearity and therefore, are difficult to analyse and predict accurately. Hence, for such data, it would not be practicable to expect a single, linear model to capture these distinct behaviours. Linear relationships and their combinations for describing the behaviour of such data are often found to be grossly inadequate. In general time series analysis, it is known that there are large numbers of nonlinear features such as cycles, thresholds, bursts, chaos, heteroscedasticity, asymmetries and combinations of one or more of these. Tong (1990), Franse, and van Dijk (2000) and Tsay (2010) have presented the various types of models that can be cast into these forms.

Nowadays, there are various applications of nonlinear time series models to different fields, such as meteorology, finance, engineering and econometrics. The nonlinear time series models have been used extensively in recent years for modeling time series data that cannot be adequately represented using linear models. Hipel and McLeod (1994) hypothesized that, although a linear model may be adequate to describe average annual river flows, the relationship between daily river flow and precipitation may be nonlinear. For examples, Tong (1990) provides an introduction to different types of nonlinear time series modeling primarily in the univariate setting. Chen and Tsay (1993, 1996) and Lewis and Ray (1997) investigated techniques for obtaining bivariate nonlinear models. Terasvirta (1993) mentioned vector nonlinear autoregressive processes, vector nonlinear average processes and multiple bilinear time series models in passing but concentrated on statistical inference for nonlinear models using parametric procedure.

Before fitting a nonlinear time series model to a given set of data, it is good if the nonlinearity characteristics of the data can be detected. There are various tests that have been suggested over the past years to distinguish linear from the nonlinear data sets. For example, Hunnich (1982) used the bispectrum test. They used the fact that the square modulus of normalized bispectrum is constant when the time series is linear. The hypothesis is based on the non-centrality of parameters of the marginal distribution of the square moduli, where n is the sample size. Yuan (2000) modified the Hunnich's test in such a way that the parameter being tested under the null hypothesis is the location parameters, such as the mean or variance.

The problem of nonlinear time series identification and modelling has attracted considerable attention for years in diverse fields such as biometrics, socioeconomics, transportation, electric power systems, and finance which exhibit nonlinear process. A good nonlinear model should be able to capture some of the nonlinear phenomena in the data. Once a model is selected, sufficiently strong evidence need to be found in the data to abandon the linear model. Therefore, good statistical and diagnostic tests are needed to determine the nonlinearity in time series data.

This work examined the performance of two nonlinearity tests in time series analysis; these are Kennan's test and F-test of nonlinearity. The power efficiency of each test wasstudied on different sample size, models and under the violation of assumption of stationarity based on simulated data and real data collected.

Stationarity

In Statistics, a stationary processis a stochastic process whose joint probability distribution does not change when shifted in time. Consequently, parameters such as the mean and variance, if they are present, also do not change over time.The most important assumption made about time series data is that of stationarity.

The basic idea of stationarity is that the probability laws that govern the behavior of the process do not change over time. In indeed, the process is statistically equilibrium. Specifically, a process $\{Y_t\}$ is said to be strictly stationary if the joint distribution of Ytis the same as that of $Y_{t-k}$for all t and k; t = 1, 2, …, k. In other words, the Y's are (marginally) identically distributed (see Jonathan and Kung-Sik, 2008). It then follows that $E(Y_t) = E(Y_{t-k})$ for all t and k so that the mean function is constant for all time. Additionally, $Var(Y_t) = Var(Y_{t-k})$ for all t and k so that the variance is also constant over time.

Linear Time Series Model

A relationship of direct proportionality that, when plotted on a graph, traces a straight line. In linear relationships, any given change in an independent variable will always produce a corresponding change in the dependent variable. For example, a linear relationship between production hours and output in a factory determines percentage of increase or decrease of the output. The concept of linear relationship suggests that two quantities are proportional to each other: doubling one causes the other to double as well.

Linear relationships are often the first approximation used to describe any relationship, even though there is no unique way to explain what a linear relationship is in terms of the underlying nature of the quantities. For example, a linear relationship between the height and weight of an object is different from a linear relationship between the volume and weight of an object. The second relationship makes more sense, but both are linear relationships, and they are, of course, incompatible with each other. Medications, especially for children, are often prescribed in proportion to weight. This is an example of a linear relationship. The linear time series modeling depends on the type of system that generates the data. Time series analysis may be Autoregressive Models, Moving Average Model or Autoregressive Moving Average Model (ARMA). However For the purpose of this research work we considered only general classes of second order auto regressive models.

Nonlinear Time series Model

Practitioners in many fields are increasingly faced with real data possessing nonlinear attributes. It is known that stationary Gaussian autoregressive models are structurally determined by their first two moments. Consequently, linear autoregressive models must be time reversible. Many real datasets are time irreversible, suggesting that the underlying process is nonlinear. Indeed, in Tong's seminal paper on threshold models, he would argue that no linear Gaussian model could explain the cyclical dynamics observed in sections of the lynx data (Tong and Lim, 1980). Furthermore, he argued that characteristics of nonlinear models, such as time irreversibility and limit cycles, mandated the development of practical nonlinear models to help resolve ongoing difficulties in real data. Tong's explanation and application of locally linear threshold models introduced striking opportunities for model building strategies.

The pioneering work in time series modeling is due to Wiener who had produced a very general class of nonlinear model, called Volterra series expansion and is generally given as follows;

$$X_t = \mu + \sum_{i=-\infty}^{\infty} \alpha_i X_{t-i} + \sum_{i,j=-\infty}^{\infty} \alpha_{ij} X_{t-i} X_{t-j} + \sum_{i,j,k=-\infty}^{\infty} \alpha_{i,j,k} X_{t-i} X_{t-j} X_{t-k} + \cdots \quad 1$$

$$X_t = \mu + \sum_{i=-\infty}^{\infty} \beta_i e_{t-i} + \sum_{i,j=-\infty}^{\infty} \beta_{ij} e_{t-i} e_{t-j} + \sum_{i,j,k=-\infty}^{\infty} \beta_{ijk} e_{t-i} e_{t-j} e_{t-k} + \ldots \quad 2$$

Where μ is the mean level of X_t, e_t, $(\infty < t < \infty)$ is a strictly stationary process of independent and identically distributed random variables. Obviously, Xt is nonlinear if one of the higher order coefficients $\alpha_{ij}, \alpha_{i,j,k} \beta_{ij}$ or β_{ijk} is non zero(Ibrahim *et al,* 2005).

For instance, the model 1 and 2 above can be illustrated with simple structures (i, j = 1, 2, k = 0) as follows;

$$X_t = \mu + \sum_{i=1}^{2} \alpha_i X_{t-i} + \sum_{i=1}^{2}\sum_{j=1}^{2} \alpha_{ij} X_{t-i} X_{t-j}$$

$$\Rightarrow X_t = \mu + \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \alpha_{11} X_{t-1}^2 + \alpha_{12} X_{t-1} X_{t-2} + \alpha_{22} X_{t-2}^2 \quad \ldots\ldots\ldots\ldots..3$$

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Most linear models can be expressed into Volterra expansion form which includes the autoregressive model of order p, [AR (p)], the moving average model of order q [MA (q)] and the autoregressive moving average model of order p and q [ARMA (p,q)].

MATERIALS AND METHODS

Several authors such as Chan and Tong(1986) and Tsay (1986) raised the issue that one nonlinearity test is not enough to detect nonlinearity in a data set. Nonetheless, it is expected that the nonlinearity test will suggest whether a data set is linear or otherwise. Thus, if any test does suggest that the data is nonlinear, we expect that a nonlinear model will improve the modeling of the data set.

Indeed, in this work, a set of data were generated from model 1-4, under the assumption of stationarity stated earlier and the two tests, Keenan and F-tests of nonlinearity were applied to see the behavior of their acceptance of nonlinearity. Thereafter, another set of data were generated under the violation of the stationarity and white noise assumptions.Each test is subjected to 500 replication simulation at different sample sizes for stationarity and Non-Stationarity data structures.Both tests are based on time domain approach and suitably applied on data generated from selected linear and nonlinear auto regressive models. Power efficiency of the tests was compared on the simulated data.

Keenan's Test

Keenan adopted the idea of Turkey one degree of freedom test for non-additivity to derive a time domain statistic. The test is motivated by similarity of Volterra expansions to polynomials, and is extremely simple, both conceptually and computationally. Assume that a time series $Y_{t,}$ t = 1, 2, .., n, can be adequately approximated by order of Volterra expansion in 1 and 2

The approximation will be linear if the second and other higher terms on the right hand side are zero. The Keenan's test procedure is as follows;

(i)Regress Yt on (1, Y_{t-1}, . . . Y_{t-m}) and calculate the fitted values(Yt), and the residuals, $(\hat{e_t})$, for t = m+1, . . ., n, and the residual sum of squares$\sum e_s^2$

(ii)Regress Yt2 on (1, Yt-1, . . . Yt-m) and calculate the residual (et) for t = m, . . ., n,

(iii) Regress $on\ \hat{e} = \mathrm{e}_{m+1}, \dots, \mathrm{e}_n$ on $\hat{\xi}_{m+1}, \dots, \hat{\xi}_n$

and obtain $\hat{\eta}\ and\ \hat{F}$from

$$\hat{\eta} = \hat{\eta}_0 \left(\sum_{t=m+1}^{n} \hat{\xi}_t^2 \right)$$

Where $\hat{\eta}_0$ is the regression coefficient, and

$$F_k = \frac{\hat{\eta}(n - 2m - 2)}{\sum_{t=m+1}^{n} \hat{\xi}_t^2 - \hat{\eta}^2}$$

Follows approximately F1,n-2M-2, , where the

degrees of freedom of associated with $\sum_{t=m+1}^{n} \hat{\xi}_t^2$ is (n-M)-M-1.

Keenan's test is based on the argument that if any of cij and other higher coefficients in 1 and 2 are non-zero, e.g c12, then this nonlinearity will be distributionally reflected in the diagnostics of the fitted linear model, if the residuals of the linear model are correlated with Yt-IYt-2. As in Turkey non additive test, Keenan's test used the aggregated quantity Y2t, the square of the fitted value of Yt based on the fitted linear model, to obtain the quadratic terms upon which the residual can be correlated. The idea is extremely valuable when the sample size is small because it only requires one degree of freedom.

F-Test

Tsay (1986) modifies Keenan's test by replacing the aggregated quantity Y2t by the disaggregated variable Yt-iYt-j, i,j = 1, …., M, where M is defined in Keenan's test. The F-test procedure is as follows:

(i) Regress Yt on (1, Yt-1, . . . Yt-m) and calculate the fitted values(Yt), and the residuals, $(\hat{e_t})$ˆ, for t = M+1, . . ., n. the regression model is denoted by

Yt = Wt $\emptyset$ + et, where,

Wt = (1, Yt-1, . . . Yt-m) and = $(\emptyset_0, \emptyset_1, \dots \emptyset_m)$T.

(ii)Regress vector Zt on (1, Yt-1, . . . Yt-m) and calculate the residuals (Xt), for t = M+1, ..., n. In this step, the multivariate regression model is Zt = WtH + Xt, where Zt is an m = dimensional vector defined by ZTt = Vech (UTtUt) with Ut = (Yt-1, . . . Yt-m), and Vech denoting the half stacking vector

(iii) Fit $\hat{e}_t = \hat{X}_t \beta + \epsilon_t, t = M + 1, \dots, n$ And define $\hat{F} = \dfrac{\dfrac{\sum \hat{X}_t^T \hat{e}_t (\sum \hat{X}_t^T \hat{X}_t)^{-1} \sum \hat{X}_t^T \hat{e}_t}{m}}{\dfrac{\sum e_t^2}{n - M - m - 1}}$

Where the summation is over t from M+1 to n. Here, $\hat{F}$ is asymptotically distributed as $F_{m,n-m-M-1}$

MODELS SELECTED FOR SIMULATION

Data is generated from several linear and nonlinear second orders of general classes of autoregressive models given below:

Model 1. AR(2): $Y_{ti} = 0.3Y_{ti-1} - 0.6Y_{ti-2} + e_t$

Model 2.TR(2): Yti = 0.3sin(Y_{ti-1}) - 0.6cos(Y_{ti-2})+ et

Model 3:EX(2): Yti = 0.3Y_{ti-2}+ exp(-0.6Y_{ti-2}) + et

Model 4: PL(2): Yt = $0.3Y_{t-1}^2 - 0.6Y_{t-2} + e_t$

$Y_{ti} \sim N(0,1)$ *and* $e_{ti} \sim N(0,1)$ *for stationary series and* $Y_{ti} \sim N(2000,20)$ *and* $e_{ti} \sim N(1000,10)$,

$t = 1,2, \ldots 50, 150$ *and* 300 $i = 1,2, \ldots, 3000$

The model 1, 2, 3 and 4 are linear, trigonometry, exponential and polynomial autoregressive models respectively with coefficients of Y_{t-1} being 0.3 and Y_{t-2} being -0.6. Simulation studies were conducted to investigate the performance of **Keenan's** and **F-test**. The hypothesis test were null hypothesis of nonlinearity against the alternative hypothesis of linearity of data. Thus, if the data is linear with α = 0.05, more than 95% of the replicates are expected to have the test statistic less than the critical values. Power of the two tests is compared for the different models, sample size and distributions to know which of the two tests is acceptably good for detecting nonlinearity for time series data generated from the given model.

Note that in autoregressive modeling, the innovation (error), et process is often specified as independent and identically normally distributed. The normal error assumption implies that the stationary time series is also a normal process; that is, any finite set of time series observations are jointly normal. For example, the pair (Y_1,Y_2) has a bivariate normal distribution and so does any pair of Y's; the triple (Y_1,Y_2,Y_3) has a trivariate normal distribution and so does any triple of Y's, and so forth. Indeed, this is one of the basic assumptions of stationary data. However, in this study, the data will be generated under white noise assumption of stationarity and when the stationarity assumption is violated for order of past responses and random error terms to see behavior of the models in each case. 3000 replications were used to stabilize models estimations at different combinations of n and models.

Selection Rule

Theaverage acceptance of linearity by each test was recorded as in table 1-4, at n=50, 150 and 300 representing small, mild and large samples respectively for each case (stationarity and Non-Stationarity) and model. The test with highest proportion of acceptance in a category is the best for that category. Note that only second order autoregressive models were considered in each case and situation

RESULTS AND DISCUSSIONS

Relative Performance of Keenan- and F-Tests on General Class of Stationary Autoregressive Cases at Different Sample Sizes

The performance of the following Keenan- and F-tests in detecting general classes of linear and nonlinear autoregressive cases were examined at sample size of 50, 150, and 300 which represent small, mild and large sample sizes respectively. The data were simulated using R statistical software following the assumption of stationarity earlier stated to fix the parameters. The parameters were fixed for each model as shown in model 1-4 to observe how the tests would accept the null hypothesis of linearity of stationary data. The white noise assumption of the error term was also observed to make the data simulated be stationary. Each created data were replicated 1000 times using TSA Package in R software.

Table 1: Empirical frequencies of rejection of the null hypothesis of linearity;
n =50 and 1000 replications. Nominal significance level, 0.05 (Stationary Data)

Model	Keenan's Test	F-Test
Model 1	0.8505	0.6198
Model 2	0.6449	0.5046
Model 3	0.6756	0.0283
Model 4	0.0033	0.0005

Table 2: Empirical frequencies of rejection of the null hypothesis of linearity;
n =150 and 1000 replications. Nominal significance level, 0.05(Stationary Data)

Model	Keenan's Test	F-Test
Model 1	0.4731	0.9280
Model 2	0.0514	0.0414
Model 3	0.0088	0.0062
Model 4	0.0012	0.0000

Table 3: Empirical frequencies of rejection of the null hypothesis of linearity;
n =300 and 1000 replications. Nominal significance level, 0.05(Stationary Data)

Model	Keenan's Test	F-Test
Model 1	0.5179	0.8706
Model 2	0.0735	0.0179
Model 3	0.0169	0.0061
Model4	0.0000	0.0000

Table 4: Effect of Sample size on the Power of the Tests for model 1-4
Nominal significance level, 0.05 (Stationary Data)

Model	Sample Size	Model 1	Model 2	Model 3	Model 4
Keenan	50	0.6198	0.6449	0.6756	0.0033
	150	0.4731	0.0514	0.0088	0.0012
	300	0.5179	0.0735	0.0169	0.0000
F-test	50	0.8505	0.5046	0.0283	0.0005
	150	0.9280	0.0414	0.0062	0.0000
	300	0.8706	0.0179	0.0061	0.0000

Relative Performance of Keenan- And F-Tests on General Class of Non-Stationary Autoregressive Cases at Different Sample Sizes

One of the objectives of this study is to find out the performance of Keenan and F-test of nonlinearity on general classes of linear and nonlinear autoregressive, simulated with violation of assumption of non-stationarity. Since most of real life data collected are non-stationary, there is need to investigate which of these tests is suitable for non-stationary data. One major assumption of stationarity is validity of white noise assumption of error term; the error term is independently distributed with zero mean and positive variance.

Indeed, in this work non-stationarity was injected in our simulated data from different models used for the simulation by violating the independence and normality assumption of error term in the following way to know the effect of non-stationarity on each model at different sample sizes: the results are displayed in table 5-8

$$Y_t \sim N(2000,20) \ and \ e_t \sim N(1000,10)$$

The value of mean and variance were specified based on the history of nature of real life data considered, Data on Nigeria Gross Domestic Products (GDP)

Table 5: Empirical frequencies of rejection of the null hypothesis of linearity;
n =50 and 1000 replications. Nominal significance level, 0.05 (Non-Stationary Data)

Model	Keenan's Test	F-Test
Model 1	0.3697	0.4607
Model 2	0.000	0.3072
Model 3	0.0000	0.3123
Model 4	0.0000	0.0034

Table 6: Empirical frequencies of rejection of the null hypothesis of linearity;
n =150 and 1000 replications. Nominal significance level, 0.05 (Non-Stationary Data)

Model	Keenan's Test	F-Test
Model 1	0.3840	0.2088
Model 2	0.0000	0.7596
Model 3	0.0000	0.3953
Model 4	0.0000	0.0017

Table 7: Empirical frequencies of rejection of the null hypothesis oflinearity;
n =300 and 1000 replications. Nominal significance level, 0.05 (Non-Stationary Data)

Model	Keenan's Test	F-Test
Model 1	0.9811	0.0819
Model 2	0.0000	0.2931
Model 3	0.0169	0.0061
Model 4	0.0000	0.0000

Table 8: Effect of Sample size on the Power of the Tests for model 1-4
Nominal significance level, 0.05 (Non-Stationary Data)

Model	Sample Size	Model 1	Model 2	Model 3	Model 4
Keenan	50	0.3697	0.0000	0.0000	0.0000
	150	0.3840	0.0000	0.0000	0.0000
	300	0.9811	0.0000	0.0000	0.0000
F-test	50	0.4607	0.3072	0.3123	0.0034
	150	0.2088	0.7596	0.3953	0.0017
	300	0.0819	0.2931	0.7670	0.0000

Table 9: Results of Nonlinearity Tests on Nigeria GDP

Test	Test Statistic	Critical value	Conclusion
Keenan's Test	10.982	0.0052	Nonlinear
F-Test	8.0797	0.0293	Nonlinear

Table 1-8 show the results of analyses of performance of Keenan's test and F-test with respect to the model 1-4 at small, mild and large sample sizes taken to be 50, 150 and 300 respectively under the assumption of stationarity and violation of the assumption of stationarity. The two tests were compared at the 5% level of significance for two tailed test in each case. The average powers of the tests for both tests were computed for easy comparison.

CONCLUSION

We noticed that both test do not reject the linearity of the Model 1, linear autoregressive at different sample sizes. However, F-test has higher power of acceptance than Keenan test when data is stationary while Keenan's test performs better for non-stationary data especially at large sample size. In model 2-4, trigonometric, exponential and polynomial auto regressive models respectively, most of the average p-value are less than the 5% level of significance and as the sample size increases the p-value decreases indicating the significant of linearity of the models by the two tests. Meanwhile, the F-test perform better as its average p-values are less than Keenan's test at the three sample sizes for stationary data and vice versa for non-stationary data as shown in the summary Table 4 and 8.

More so, both tests wrongly accept the null hypothesis of linearity for model 2, 3, and 4 with their average p-values greater than 5% level of significance at sample size 50 for stationary data. When the Non-Stationarity was introduced in data generated F-test's p-value were greater than 5% and therefore wrongly accept the null hypothesis of linearity of nonlinear autoregressive model except that of polynomial model which its linearity was rightly rejected and more powerful when the sample size increases. While Keenan's test has the p-value close to zero which show the significant of linearity of the three nonlinear models at the three sample sizes and therefore considered as the most powerful test for non-stationary data

Finally, we provided illustrative examples by applying the statistical tests to real life datasets and results obtained are desirable.

REFERENCES

Chan, W. S., and Tong, H. (1986), On Tests for Non-linearity in Time Series Analysis. *Journal of Forecasting,*5: 217-228.

Chen R. and Tsay, R.S. (1993),Nonlinear additive ARX models. *J. Amer. Statist.Assoc.* 88, 995-967.

Chen R. and Tsay, R.S. (1996). Nonlinear transfer functions. *Journal of Nonparametric Statistics,* 6:193-204.

Franse , P.H. and van Dijk, D. (2000). Nonlinear error-correction models for interest ates in the Netherlands, inW.A.

Hunnich, M. J. (1982), Testing for Gaussianity and Linearity of a Stationary Time Series. *Journal of Time Series Analysis*. 3(3): 169-176.

Hipel, K. W. and McLeod, A.I. (1994). Time Series Modeling of Water Resources and Environmental Systems. Amsterdam: *Elsevier.*

Ibrahim M., Azami Z. and Mohd., S.Y.(2005), *JurnalTeknologi*, 42(C), 1–10 Universiti Teknologi Malaysia

Jonathan D. C. and Kung-Sik C. (2008), Time Series Analysis with Applications in R Second Edition, Springer Science+Business Media, LLC

Keenan, D.M. (1985). A Tukey non-additivity-type test for time series nonlinearity. *Biometrika* 72, 39-44.

Lewis, P., and B. Ray (1997): "Modeling nonlinearity, long-range dependence, and periodic phenomena in sea surface temperatures using TSMARS, "*Journal of the American Statistical Association,* 92: 881-893.

Terasvirta, T. (1993): Testing linearity against smooth transition autoregressive models.*Biometrika* 74, 491-499.

Tong, H. (1990): Nonlinear TimeSeries: A Dynamical System Approach, Oxford: Clarendon.

Tong H. and Lim, K. S. (1980). Threshold Autoregression, Limit Cycles and Cyclical Data. *Journal of Royal Statist. Soc*., B 42:245-292.

Tsay, R.S. (1986). Nonlinearity tests for time series. *Biometrika* 73, 461-466.

Tsay, R.S. (1998). Testing and modeling multivariate threshold models. *J. Amer. Statis. Assoc*.,93, 1188- 1202.

Tsay, R. S. (2010): Analysis of Financial Time Series; 3rd Edition, Published by John Wiley & Sons, Inc., Hoboken, New Jersey. Published simultaneously in Canada

Yuan, J. (2000): Testing Linearity for Stationary Time series using the Sample Inter-quartile Range. *Journal of Time series Analysis* 21(6) 714-722

Yule, G. U. (1926). "Why do we sometimes get nonsense-correlations betweentime-series? —Astudy in sampling and the nature of time-series." *Journal of theRoyal Statistical Society,* 89:1, 1–63.