FULafia Journal of Science & Technology
ISSN (print) 2449-0954 \ ISSN (online) 2636-4972
https://lafiascijournals.org.ng/index.php/fjst Sponsored By TERTIARY EDUCATION TRUST FUND
Manuscript => Received: Feb., 2025; Accepted: May, 2025; Published: Aug., 2025 https://doi.org/10.62050/fjst2025.v9n2.498

THERMAL RADIATION EFFECT ON A BOUYANCY-INDUCED COUETTE FLOW DUE TO NEWTONIAN HEATING IN AN UPSTANDING CHANNEL

Godwin Ojemeri[1]*, Muhammed Murtala Hamza[2], Agnes Marian Ochiba[2], Isaac Obiajulu Onwubuya[3], Adeniran Kolade Ademuwagun[4], Abubakar Muhammad Tsafe[2], Ibrahim Ahmad Sifawa[5]
[1]*Department of Mathematics, Federal University of Agriculture, Zuru, PMB 28, Kebbi State*
[2]*Department of Mathematics, Usmanu Danfodiyo University, PMB 2346, Sokoto State*
[3]*Department of Mathematics, Air Force Institute of Technology, PMB 2104, Kaduna State*
[4]*Department of Electrical and Electronics, Air Force Institute of Technology, PMB 2104, Kaduna State*
[5]*Department of Mathematics, Sokoto State University, Sokoto State*
*Corresponding email: godwinojemeri@gmail.com

ABSTRACT
This paper illustrates the impact of optically thick thermal radiation on buoyancy-induced flow with viscous dissipation and Navier slip condition over an up-facing channel. The modelled equations are nonlinear coupled ordinary differential equations, which are treated with the regular perturbation method. The actions of key parameters controlling the flow behaviour in terms of momentum and energy distributions are demonstrated graphically. The skin friction and Nusselt number on the both surfaces have also been computed. The present study is valid for the limiting case because it is based on comparison with earlier studies that back it up. The significant results from this study are: thermal radiation R act as extra aiding force, i.e., growing values of R boosts the fluid temperature and velocity, greater Brickman number represent better convective heating at the channel surface, leading to a stronger temperature and velocity. Additionally, the heat transfer rate weakens as boundary thickness causes less heat transfer gradient. The outcome of this research will contribute significantly in widening the applications of thermal radiation effects for multiple heating devices and industrial uses for energy production in solar systems, wound treatment in medical science, space vehicles and aircraft propulsion in engineering and space technology, to mention a few.

Keywords: Free convection, viscous dissipation, thermal radiation, Navier slip, thermal buoyancy effect, vertical channel

INTRODUCTION
Natural convection in a boundary layer cavity is a flow that results from the interaction of gravity with density changes within a fluid. These changes are evident because of the temperature or concentration components or due to their interplay. Studies pertaining free convection flows in vertical parallel plate channels for a single phase of an incompressible viscous fluids have received much attention in recent years both theoretically (exact or approximate solutions) and experimentally owing to the fact that many practical applications involve natural convection heat transfer (Al-Subaie & Chamkha, 2004; Zullkifree *et al*., 2019). The laminar flow of a viscous fluid along two parallel walls, one of which moves tangentially with respect to the other, is referred to as Couette flow. This flow can be motivated by a pressure gradient that is present in the flow direction (Mng'ang'a, 2023). A number researchers have accomplished their works on the above subject. Khan *et al*. (2022) analyzed the second law to determine how Newtonian heating affects the Couette flow of a viscoelastic dusty fluid and the transfer of heat in a rotating frame. Joseph *et al*. (2014) investigated heat transfer in an inclined magnetic field with unsteady MHD Couette flow between two infinitely parallel porous plates. The temperature-dependent transient MHD Couette flow and heat transfer of dusty fluid were studied by Mosayebidorcheh *et al*. (2015). Jha *et al*. (2015) investigated thermal radiation with unsteady MHD free convective Couette flow. Raju *et al*. (2016) discussed the influence of diffusion thermo and thermal diffusion on a natural convection Couette flow using the finite element method. Job and Gunakala (2016) described the impact of radiation factor over an upstanding porous walls using Galerkin's finite element scheme. Also, Ali *et al*. (2017) analyzed the Couette flow of a Maxwell fluid in three dimensions with periodic injection/suction. Hussain *et al*. (2018) analyzed the instability of the MHD Couette flow of an electrically conducting fluid. Anyanwu *et al*. (2020) discussed the influence of radioactive and a constant pressure gradient on an unsteady MHD Couette flow. Ajibade *et al*. (2021) interpreted the influence of viscous dissipative fluid on steady free convection Couette flow over a vertical channel affected by heat source factor. Recently, unsteady convective Couette flow with heat sink, radiation, heat source, magnetic field and thermal property effects was extensively investigated in the works of Omokhuale and Jabaka (2022a, 2022b) and Omokhuale and Jabaka (2023).
The concept of free convection flows in parallel plate is usually modelled under the assumptions of constant surface temperature, ramped wall temperature, or constant surface heat flux (Rajput 2011a, 2011b; Narahari 2012; Singh & Sarehu 2015). Although, in

most practical scenarios where the heat transfer from the surface is assumed to be proportional to the local surface temperature, the initial assumptions does not hold. This types of flows are termed as conjugate convective flows, and the proportionally condition of the heat transfer to the local surface temperature is termed as Newtonian heating (Lesnic *et al*., 2004). The natural convection boundary layer flow over a vertical surface with Newtonian heating was first studied by Merkin (1994). In view of this, Hamza *et al*. (2025a, 2025b) recently investigated the impact of nonlinear radiation on unsteady free and mixed bioconvections with nonlinear density variation over some nanoparticles affected by Newtonian heating using legendary special collocation approach. Swain *et al*. (2020) analytically and numerically presented the study of magnetized flow of a Newtonian fluid affected by Newtonian heating over a stretching sheet in the presence of porous material. Shehzad *et al*. (2014) investigated three-dimensional flow of Jeffrey fluid with Newtonian heating. Hamza (2016) emphasized the influence of Navier Slip and Newtonian heating in transient flow of an exothermic fluid in a vertical channel. Das *et al*. (2015) performed a numerical analysis on unsteady heat and mass transfer of hydro-magnetic Casson fluid across a vertical plate in the involvement of chemical reaction and thermal radiation. Qayyum *et al*. (2017) described the heat and mass transfer of water-B nanofluid across a stretching sheet in the coexistence of chemical reaction and thermal radiation. Hayat *et al*. (2018) discussed the impact of MHD flow of Powell-Eyring fluid by a stretching cylinder with thermal radiation through an inclined magnetic field under the Newtonian heating effect. Zin *et al*. (2018) presented an exact and numerical solution of unsteady MHD heat and mass transfer flow of Jeffrey fluid flowing through an oscillatory vertical plate provoked by thermal radiation and Newtonian heating factors.

The internal heat emission as a result of viscous forces in molecular fluid-particle interplay is termed as viscous dissipation, and the mechanical heat dissipated in this form profoundly affects the thermal buoyancy and thermal characteristics of a fluid motion. Therefore, the presence of viscous dissipation is crucial in most lubrication industries. These kind of fluid flow has gotten a lot of attention because of its applications in science, engineering, and industrial technologies such as gas drainage, cooling of electrical appliances, geothermal energy, plasma physics, gas turbines, fossil fuel combustion, food processing industries, and so on (Ajibade *et al*. 2021). One of the earliest works that considered viscous dissipation was the work of Gebhart (1962), which used a perturbation method to investigate the effect of viscous dissipation on natural convection. Over the years, several authors have exemplified their work on this field under consideration. Omokhuale and Ojemeri (2024) used homotopy perturbation method to analyze the influence of viscous dissipation on a buoyancy-induced Couette flow in a vertical channel affected by Newtonian heating. It is concluded from their results that viscous dissipation and Newtonian heating parameters are envisaged to promote the fluid velocity. Isa *et al*. (2024) numerically investigated the impact of unsteady magnetized free convection heat transfer flow of viscous dissipation in a vertical channel in the presence of Newtonian heating using the Implicit Finite Difference Method (IFMD). They concluded that, as time increases, rising level of the viscous dissipation parameter is seen to improve the fluid velocity. Yale *et al.* (2023) investigated the impacts of superhydrophobicity and MHD on a viscous dissipative fluid in a slit microchannel using regular perturbation approach. Onwubuya *et al*. (2024) recently evaluated the impact of viscous dissipation, porous medium and superhydrophobicity on the free convection of an electrically conducting fluid over an upstanding microchannel affected by an imposed magnetic field. The plates were alternatively heated and incorporated with a heat source/sink effect. Omokhuale *et al*. (2024a, 2024b)underscored the significance of viscous dissipation on both free and mixed convection flows of a nanofluid flow along a semi-infinite flat plate in an oscillating system, taking into account the impacts of MHD, chemical reaction, porous medium and heat sink. Uka *et al.* (2023) analytically and numerically solved the problem of fluid flow in the presence of viscous dissipation and permeability effect as a result of a wedge in motion using a well-established regular perturbation and Mathematica V.10 scheme. Amar *et al*. (2023) investigated the impacts of viscous dissipation, thermal radiation on the MHD heat transfer flow of Casson fluid across a moving wedge with convective boundary condition in the existence of and internal heat generation/absorption. Other studies that considered the effect of viscous dissipation on fluid flow are the works of Ajibade and Umar (2020a, 2020b), Ojemeri and Onwubuya (2023), Prakash and Sivakumar (2018), among others.

From the preceding discourse, the interplay of optically thick thermal radiation with viscous dissipation affected by Navier slip conditions has not been exhaustively discussed. Therefore, the intention of this paper is to contribute to the existing works by investigating the influence of steady free convection flow of an incompressible viscous dissipative fluid affected by thermal radiation in a convectively heated plate using the work of Omokhuale and Ojemeri (2024) as a benchmark. The modeled ordinary differential equations are treated with the regular perturbation method against the homotopy perturbation method (HPM) used in earlier work. This article will not only serve as a generalization of Omokhuale and Ojemeri (2024)'s work but also can be termed as a comparative study. It is believed that the outcomes of this study will contribute significantly in widening the applications of thermal radiation effects for multiple heating devices and industrial uses for energy production in solar system, wound treatment in medical science, space vehicles and aircraft propulsion in engineering and space technology, etc.

Formulation of the Problem

Consider a steady buoyancy-induced flow of an incompressible viscous fluid passing through two vertical parallel plates affected by Newtonian heating and heat radiation effects. As shown in Figure 1, the wall at $y_0 = 0$, is instigated by Navier slip effect while the no-slip surface is kept at $y_0 = H$. The flow is subjected to the presence of thermal buoyancy effects. All the fluid properties are assumed to be constants. The flow variables are functions of space y only. In view of the present formulation, the following assumptions are further made:

i. The fluid is laminar, viscous and incompressible.
ii. The flow is fully developed and obeys the Bousinesq's approximation.
iii. It is assumed that the fluid is subjected to thermal buoyancy effect.
iv. The fluid is assumed to be affected by thermal radiation.

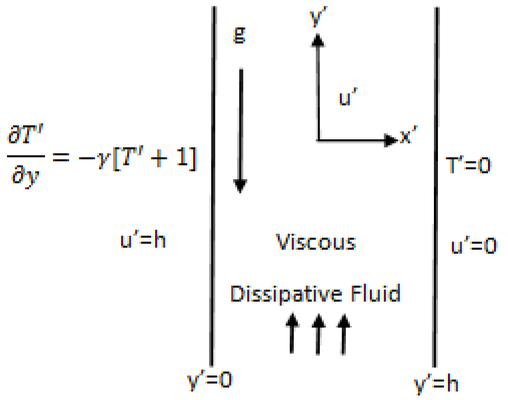

Figure 1: Flow geometry and physical coordinates

Following Omokhuale and Ojemeri (2024) and employing the Boussinesq buoyancy and Rosseland approximations with the appropriate boundary conditions, the dimensional governing equations for the present problem can be expressed as:

$$v\frac{d^2u'}{dy'^2} + g\beta(T' - T_0) = 0 \qquad (1)$$

$$\frac{k}{\rho c_p}\frac{d^2T'}{dy'^2} - \frac{1}{\rho c_p}\frac{dq_r}{dy'} + \frac{v}{c_p}\left(\frac{du'}{dy'}\right)^2 = 0 \qquad (2)$$

Subject to the boundary conditions:

$$\left.\begin{array}{l} \left.\begin{array}{r} u' = U_0 \\ \frac{dT'}{dy'} = -\gamma[T' + 1] \end{array}\right\} at\ y' = 0 \\ \left.\begin{array}{r} u' = 0 \\ T' = T_h^0 \end{array}\right\} \quad at\ y' = h \end{array}\right\} \qquad (3)$$

Applying the Rosseland approximation (Raptis, 1998), the radiative heat flux qr is indicated by:

$$qr = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \qquad (4)$$

Now according to Raptis (1999), it is assumed that temperature difference inside the flow is sufficiently meagre so that T^4 may be considered as a linear function of temperature in a Taylor series about T_0 and ignoring higher terms one can get:

$$T^4 \cong 4T_0^3T - 3T_0^4 \qquad (5)$$

Using equations (4) and (5) equation (6) becomes:

$$\left(1 + \frac{16\sigma^*T^3}{3kk^*}\right)\frac{\partial^2 T}{\partial y^2} - Br\left(\frac{du}{dy}\right)^2 = 0 \qquad (6)$$

Where $R = 16\sigma^*T^3/3kk^*$ is taken as the radiation parameter, so the above equation becames

$$(1 + R)\frac{\partial^2 T}{\partial y^2} - Br\left(\frac{du}{dy}\right)^2 = 0 \qquad (7)$$

And where the dimensionless parameters used are:

$$u = \frac{u'}{U}, y = \frac{y'}{h}, \theta = \frac{T' - T_0}{T_w - T_o}, x = \frac{x'v}{Uh^2}, B_r = \frac{hH}{k}, Gr = \frac{g\beta(T_w - T_0)h^3}{v^2} \qquad (8)$$

Using the dimensionless quantities in eqn (8), the basic eqns (9) to (10) becomes:

$$\frac{d^2U}{dy^2} + Gr\theta = 0 \qquad (9)$$

$$(1 + R)\frac{d^2\theta}{dy^2} + Br\left(\frac{du}{dy}\right)^2 = 0 \qquad (10)$$

While the boundary conditions in dimensionless forms becomes:

$$\left.\begin{array}{l} \left.\begin{array}{r} \frac{d\theta}{dY} = -\gamma(\theta + 1) \\ U = 1 \end{array}\right\} at\ y = 0 \\ \left.\begin{array}{r} \theta = 0 \\ U = 0 \end{array}\right\} at\ y = 1 \end{array}\right\} \qquad (11)$$

Where Gr is thermal Grashof number, R is the thermal radiation, Br = EcPr is the Brinkman number, representing the viscous dissipation and γ is the Navier slip parameter.

Solution Procedure

The coupled nonlinear governing equations is solved using regular perturbation method such that the solutions of the temperature and velocity is assumed as;

$$U = u_0 + \mathrm{Br}(u_1) \qquad (12)$$

$$T = t_0 + \mathrm{Br}(t_1) \qquad (13)$$

Inserting eqns (12) and (13) into eqns (9) to (11), and taking the coefficients of Br^0 and Br, we derive the following system of ordinary differential equations with their corresponding boundary conditions as:

$$\mathrm{Br}^0: \frac{d^2u_0}{dy^2} + Gr\theta_0 = 0 \qquad (14)$$

$$\mathrm{Br}: \frac{d^2u_1}{dy^2} + Gr\theta_1 = 0 \qquad (15)$$

$$\mathrm{Br}^0: (1 + R)\frac{d^2\theta_0}{dy^2} = 0 \qquad (16)$$

$$\mathrm{Br}: (1 + R)\frac{d^2\theta_1}{dy^2} + \left[\frac{du_0}{dy}\right]^2 = 0 \qquad (17)$$

$$\left.\begin{array}{r} \mathrm{Br}^0: \frac{d\theta_0}{dy} = -\gamma(\theta_0 + 1) \\ \mathrm{Br}: \frac{d\theta_1}{dy} = -\gamma\theta_1 \\ \mathrm{Br}^0: u_0 = 1 \\ \mathrm{Br}: u_1 = 0 \end{array}\right\} \text{at } y = 0 \qquad (18)$$

and

$$\left.\begin{array}{r} \mathrm{Br}^0: \theta_0 = 0 \\ \mathrm{Br}: \theta_1 = 0 \\ \mathrm{Br}^0: u_0 = 0 \\ \mathrm{Br}: u_1 = 0 \end{array}\right\} \text{at } y = 1 \qquad (19)$$

The solutions of temperature & velocity is obtained as:

$$\theta_0 = a_1 L_1 y + a_1 L_2 \quad (20)$$

$$u_0 = -Gr\left(a_1 L_1 \frac{y^3}{6} + a_1 L_2 \frac{y^2}{2}\right) - L_3 y - L_4 \quad (21)$$

$$\theta_1 = -Gr^2 a_1^3 L_1^2 \frac{y^6}{120} - Gr^2 a_1^3 L_1 L_2 \frac{y^5}{20} - Gra_1^2 L_1 L_3 \frac{y^4}{12} - Gra_1^2 L_2 L_3 \frac{y^3}{3} - Gr^2 a_1^3 L_2^2 \frac{y^4}{12} - a_1 L_1^2 \frac{y^2}{2} + L_5 y + L_6 \quad (22)$$

$$u_1 = Gr^3 a_1^3 L_1^2 \frac{y^8}{6720} + Gr^3 a_1^3 L_1 L_2 \frac{y^7}{840} + Gr^2 a_1^2 L_1 L_3 \frac{y^6}{360} + Gr^2 a_1^2 L_2 L_3 \frac{y^5}{60} + Gr^3 a_1^3 L_2^2 \frac{y^6}{360} + Gra_1 L_3^2 \frac{y^4}{24} - GrL_5 \frac{y^3}{6} - GrL6y23+L7y+L8 \quad (23)$$

The rate of heat transfer and skin friction at both surfaces is also determined as:

$$\left.\frac{d\theta}{dy}\right|_{y=0} = a_1 L_1 + \mathrm{Br} L_5 \quad (24)$$

$$\left.\frac{d\theta}{dy}\right|_{y=1} = a_1 L_1 + \mathrm{Br}(-6a_2 - 5a_3 - 4a_4 - 3a_5 - 4a6-2a7+L5) \quad (25)$$

$$\left.\frac{dU}{dy}\right|_{y=0} = -L_3 + \mathrm{Br} L_7 \quad (26)$$

$$\left.\frac{dU}{dy}\right|_{y=1} = -Gr\left(\frac{a_1 L_1}{3} + a_1 L_2\right) - L_3 + \mathrm{Br}(8a_{14} + 7a_{15} + 6a16+5a17+6a18+4a19-3a20-2a21+L7) \quad (27)$$

Where all the constants used are indicated in the appendix

RESULTS AND DISCUSSION

The problem of viscous dissipative Couette flow of a thermal buoyancy factor moving vertically within two parallel plates is investigated in the presence of Navier slip and thermal radiation impacts. Regular perturbation approach is used to solve the resultant modelled equations. Graphs have been plotted to illustrate the consequences of major parameters dictating the flow pattern. The reference values selected for this current analysis are (R=0.1, γ = 0.01, Br=0.001 and Gr=0.1).

The deviation of the radiation parameter R on the temperature gradient, as other parameters values are fixed, is demonstrated in Figure 3. It is obvious from this figure that the temperature of the fluid is substantially boosted with a rise in R. This scenario happens because the radiation parameter thickens the thermal boundary as well as allowing the fluid to produce heat energy from the flow domain to make the system settles down. In Figure 4, the fluid velocity is rising as a result of higher thermal radiation effect, which is due to heat generation that uplifts the flow speed. Additionally, the heat emitted from the heated wall strengthens the fluid particles. Similar behavior is demonstrated in the work of Hamza *et al*. (2023), establishing the correctness of the present investigation. Figures 5 and 6 describe the function of viscous dissipation Br on steady temperature and velocity gradients. Since Eckert number emanates from the kinetic energy of flow and heat enthalpy changes, an increase in Eckert number promotes the kinetic energy. Consequently, higher viscous dissipative factors boost fluid's temperature and velocity, respectively. Again, it is true that rising levels of Brickman number represent better convective heating at the channel surface, leading to a stronger temperature and velocity at the plate surfaces. Makinde and Aziz (2011) assert that raising the local Brinkman value causes the convective heating at the lower channel wall. As a result of this, higher surface temperatures is produced that penetrate deeper into the still fluid. The present result coincides with those obtained by Omokhuale and Ojemeri (2024) and Onwubuya *et al*. (2024). Figures 7 and 8 demonstrate the impact of the Navier slip parameter γ on the energy and fluid motion coefficients. It is worthy of note that mounting value of γ is envisaged to reduce the density of the fluid due to the enlargement of the velocity boundary surface thickness. As a result, stronger convective heating scenario encourages the velocity of the fluid. This finding is in agreement with Zulkifree *et al*. (2019) and Omokhuale and Ojemeri (2024). The action of R against Br on the heat transfer rate is plotted in Figure 9a & b. It is evident that the rate of heat transfer demonstrates a decelerating tendency at both walls ($y = 0$ and $y = 1$) in the presence of R. Figure 10a & b reveal the effect of varying γ versus Br on Nusselt number. It is shown that the application of γ is seen to lower the amount of heat transfer at both plates in the channel. Figure 11a & b depict the function of R affected by Br on the skin friction coefficient. It was revealed that growing values of R promote the frictional force at the lower plate $(y = 0)$ whereas a counter behavior is noticed at the upper plate $(y = 1)$. The action of γ at both upstanding walls for skin friction is elucidated in Figure 12a & b. At the wall (y=0), there is a remarkable strengthening in the sheering stress, as demonstrated in Figure 12a, however, at y=1, the contrary effect occurs as plotted in Figure 12b.

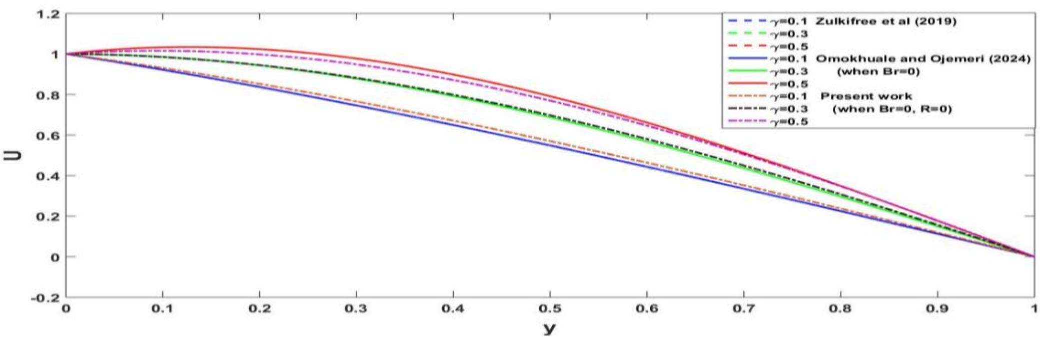

Figure 2: Comparison of the present work with previously published works

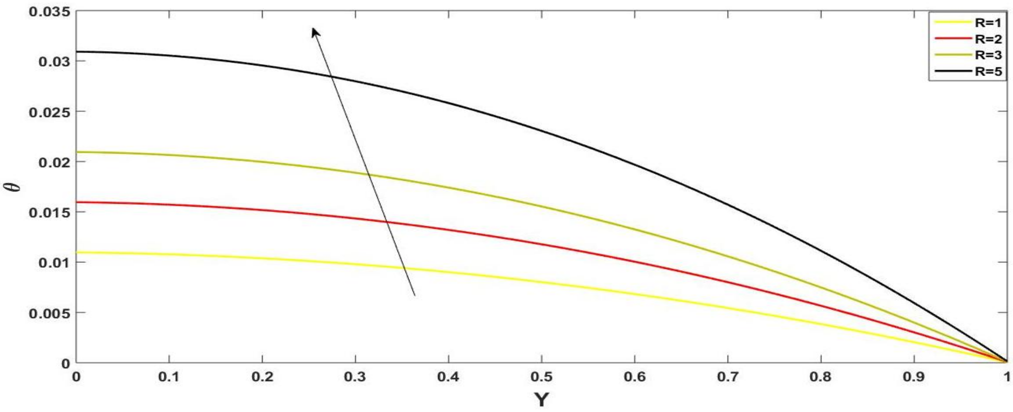

Figure 3: Result of R on temperature gradient

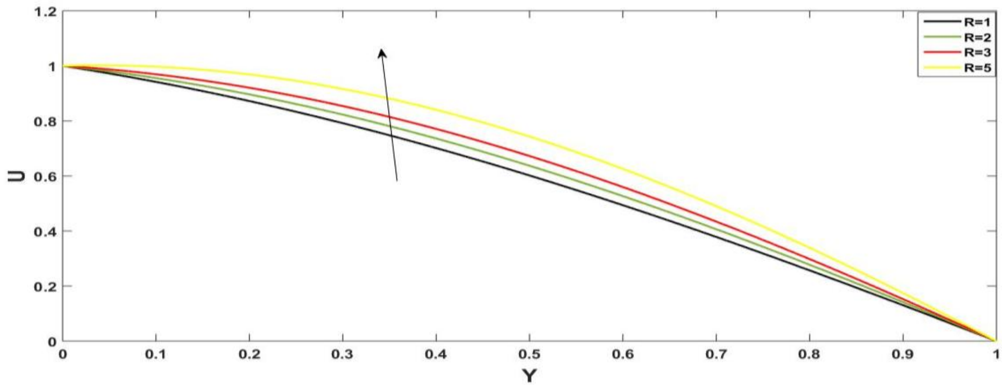

Figure 4: Result of R on velocity gradient

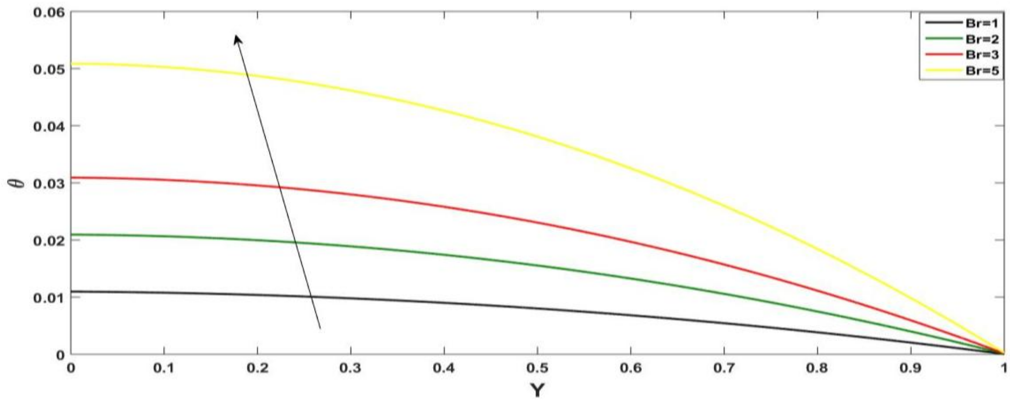

Figure 5: Result of Br on temperature gradient

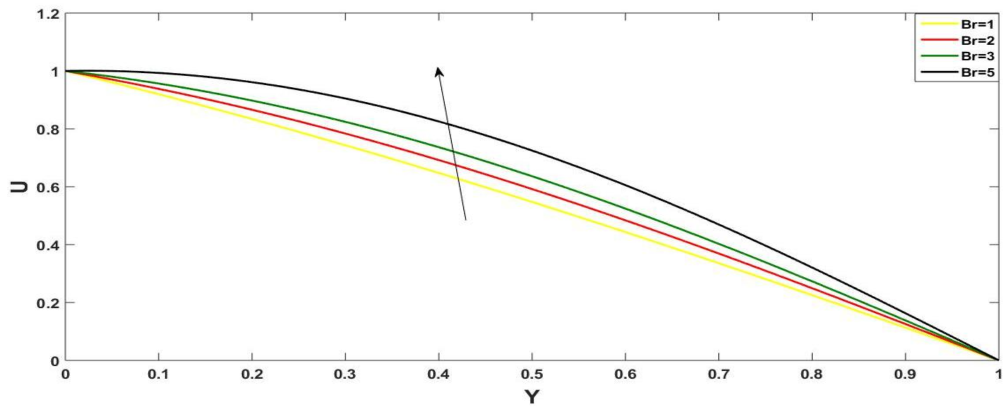

Figure 6: Result of Br on velocity gradient

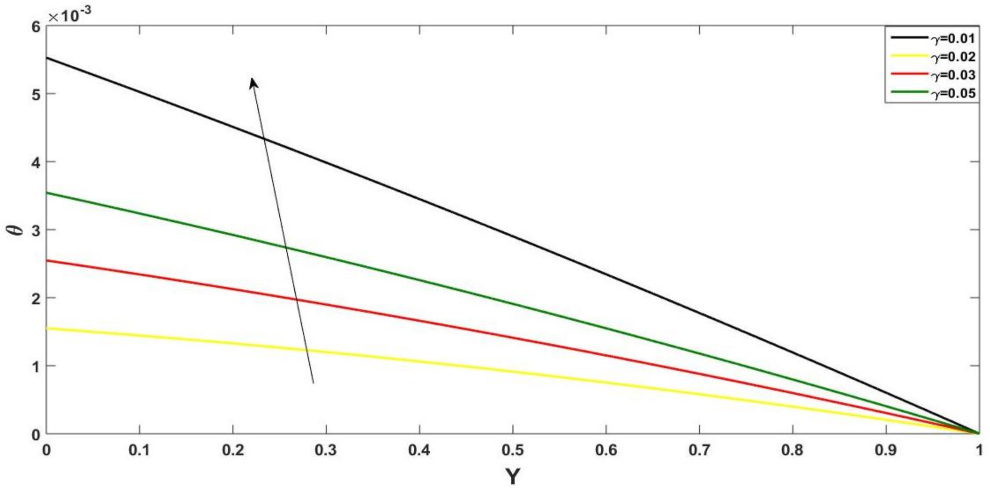

Figure 7: Result of γ on temperature gradient

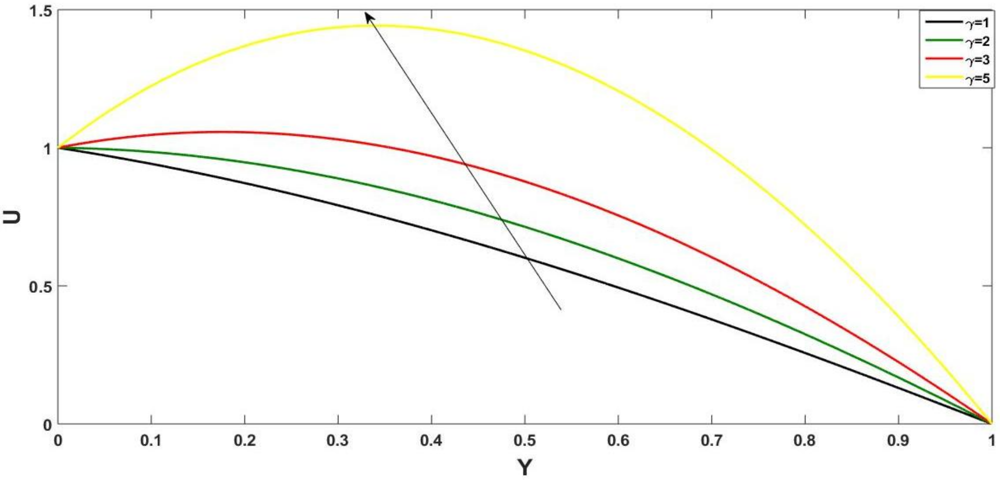

Figure 8: Result of γ on velocity gradient

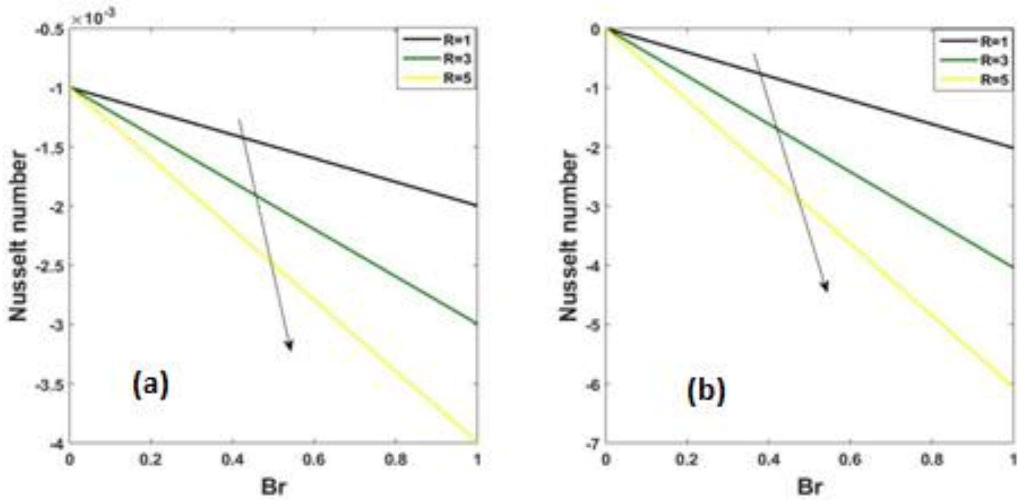

Figure 9: Nusselt number for R against Br

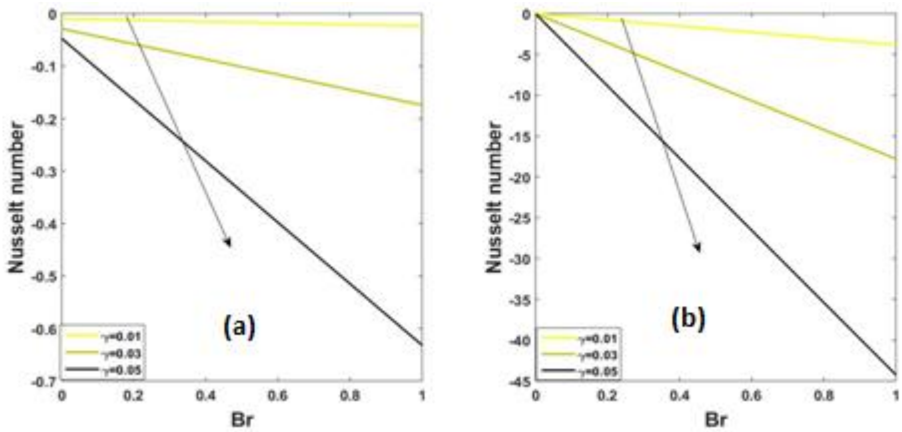

Figure 10: Nusselt number for γ against Br

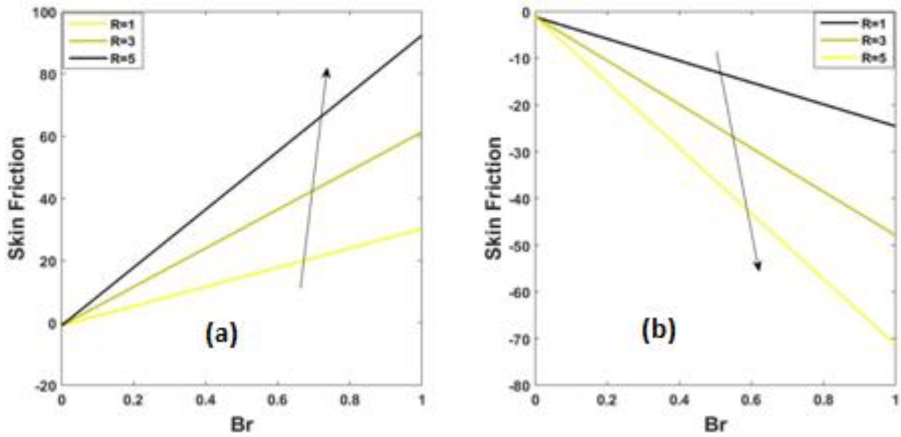

Figure 11: Skin friction for R against Br

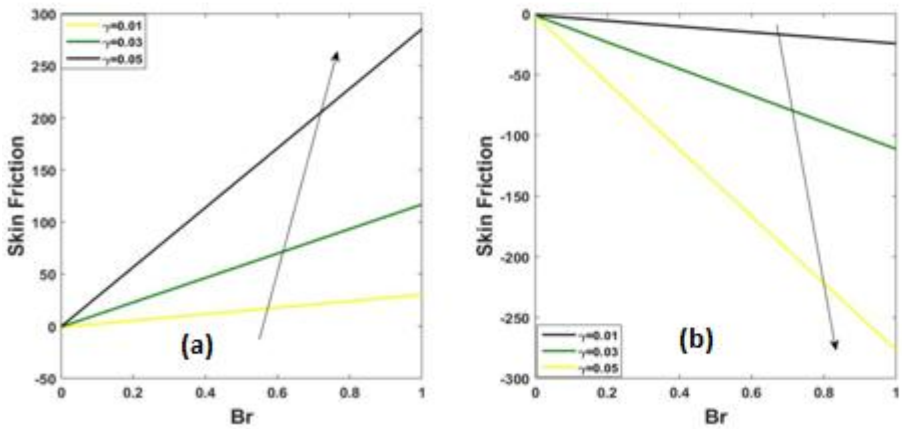

Figure 12: Skin Friction for γ against Br

Validation of results

Figure 2 have been plotted to demonstrate the relationship between the present work and previously published works. The comparison illustrates a good concurrence.

CONCLUSION

This research investigated the impacts of thermal radiation and viscous dissipation on steady fully developed Couette flow influenced by thermal buoyancy force in a convectively heated plate using the regular perturbation method. The presence of radiation and viscous dissipation is expounded thoroughly. A closed form solution has been derived from the coupled nonlinear ordinary differential equations representing the temperature and velocity components. Additionally, the physical components of engineering concerns like Nusselt number and sheer stress have also been computed. The influence of key parameters involved in the fluid is demonstrated with the aid of line graphs. For the limiting case ($R = 0$), our results are compared with the HPM solutions (Omokhuale & Ojemeri 2024), and a very good concurrence exist. Some interesting findings from this paper are outlined below:

(i) Raising levels of thermal radiation is observed to enhance the temperature and velocity of the fluid.
(ii) The effect of viscous dissipation is envisaged to improve the fluid's temperature and velocity due to the stored energy as a result of dissipation.
(iii) The fluid's velocity and temperature are substantially increased by elevating the Navier slip parameter.
(iv) The amount of heat transfer is reduced at the both walls in favor of radiation and Navier slip parameters respectively.
(v) It was concluded that higher values of radiation and Newtonian heating parameters improve the skin friction, respectively at the plate (y = 0), while a contrast behaviour happens at the plate (y = 1).
(vi) In the future, the effects of variable thermal conductivity and magnetic field would be studied on this model.

REFERENCES

Ajibade A. O., Umar A. M. and Kabir T. M. (2021). An analytical study on effects of viscous dissipation and suction/injection on a steady MHD natural convection Couette flow of heat generating/absorbing fluid. *Advances in Mechanical Engineering*, 13(5), 1–12.

Ajibade, A. O and Umar A. M. (2020a). Steady natural convection Couette flow with wall conduction and thermal boundary condition of third kind. *ZAMM Zeitschrift fur Angew and Mathematik und Mechanik*, 100, 1–18. https://doi.org/10.1002/zamm.201900095.

Ajibade, A. O. and Umar, A. M. (2020b). Effects of viscous dissipation and boundary wall thickness on steady natural convection Couette flow with variable viscosity and thermal conductivity. *Int. J. Thermo. Fluids*, 8, 100052. https://doi.org/10.1016/j.ijft.2020.100052.

Ali, Y., Rana, M. and Shoaib, M. (2017). Magneto-hydrodynamic three-dimensional Couette flow of a Maxwell fluid with periodic injection/suction. *Mathematical Problems in Engineering*, Article ID 1859693.

Al-Subaie, M. A. and Chamkha, A. J. (2004). Transient natural convection flow of a particulate suspension through a vertical channel. *Heat and Mass Transfer*, 40(9), 707–713.

Amar, N., Kishan, N. and Goud, B. S. (2023). Viscous dissipation and radiation effects on MHD heat transfer flow of Casson fluid through a moving wedge with convective boundary condition in the existence of internal heat generation/absorption. *Journal of Nanofluids,* 12, 643–651.

Anyanwu, E. O., Olayiwola, R. O., Shehu, M. D. and Lawal, A. (2020). Radiative e1ects on unsteady MHD Couette flow through a parallel plate with constant pressure gradient. *Asian Research Journal of Mathematics*, 1–19.

Das, M., Mahato, R. and Nandkeolyar, R. (2015). Newtonian heating effect on unsteady hydromagnetic casson fluid flow past a flat plate with heat and mass transfer. *Alexandria Engineering Journal*, 54(4), 871–879.

Gebhart, B. (1962), Effects of viscous dissipation in natural convection. *J. Fluid. Mech.*, 14, 225–232. https://doi.org/10.1017/S0022112062001196

Hamza, M. M. (2016). Free convection slip flow of an exothermic fluid in a convectively heated vertical channel. *Ain Shams Engineering J.*, http://dx.doi.org/10.1016/j.asej.2016.08.011

Hamza, M. M., Bello, I., Mustapha, A., Usman, U. & Ojemeri, G. (2023). Determining the role of thermal radiation on hydro-magnetic flow in a vertical porous super-hydrophobic microchannel. *Dutse Journal of Pure and Applied Sciences*, 9(2b), 297-308.

Hamza, M. M., Sheriff, A., Isah, B. Y., Bello, A. (2025a). Nonlinear thermal radiation effects on bioconvection nanofluid flow over a convectively heated plate. *International Journal of Nonlinear Mechanics*, 171, 105010.

Hamza, M. M., Sheriff, A., Isah, B. Y., Bello, A. (2025b). Nonlinear mixed convection flow of nanofluid film in a convectively heated channel, *Nonlinear Science*, 2, 100011.

Hayat, T., Hussain, Z., Farooq, M. and Alsaedi, A. (2018). Magnetohydrodynamic flow of powelleyring fluid by a stretching cylinder with Newtonian heating. *Thermal Science*, 22(1B), 371–382.

Hussain, Z., Hussain, S., Kong, T. and Liu, Z. (2018). Instability of MHD Couette flow of an electrically conducting fluid. *AIPAdvances*, 8(10), Article ID 105209.

Isa, B. U., Yale, I. D., Sarki, M. N. and Hamza, M. M. (2024). Effects of viscous dissipative fluid on Couette flow in a vertical channel due to Newtonian heating. *International Journal of Science for Global Sustainability*, 10(1), 76-82. https://doi.org/10.57233/ijsgs.v10i1.595

Jha, B. K. Isah, B. and Uwanta, I. (2015). Unsteady MHD free convective Couette flow between vertical porous plates with thermal radiation. *J. of King Saud University Sci.*, 27(4), 338–348.

Job, V. M. and Gunakala, S. R. (2016). Unsteady MHD free convection Couette flow between two vertical permeable plates in the presence of thermal radiation using galerkin's finite element method. *International Journal of Mechanical Engineering*, 2, 99–110.

Joseph, K. Daniel, S. and Joseph, G. (2014). Unsteady MHD Couette flow between two in6nite parallel porous plates in an inclined magnetic 6eld with heat transfer. *Int. Journal of Mathematics and Statistics Invention*, 2, 103–110.

Jumann eMng'ang'a (2023). Effects of chemical reaction and joule heating on MHD generalized Couette flow between two parallel vertical porous plates with induced magnetic field and Newtonian heating/cooling. *Int. J. of Mathematics and Mathematical Sci*., Article ID 9134811, https://doi.org/10.1155/2023/9134811

Khan D., Kumam, P. A., Rahman *et al*. (2022). The outcome of Newtonian heating on Couette flow of viscoelastic dusty 5uid along with the heat transfer in a rotating frame: second law analysis. *Heliyon*, 8(9), Article ID e10538.

Merkin, J. H. (1994). Natural convection boundary layer flow on a vertical surface with Newtonian heating. *International Journal for Heat and Fluid Flow*, 15, 392-398.

Mosayebidorcheh, S., Mosayebidorcheh, T., Hatami, M., Ganji, D. D. and Mirmohammadsadeghi, S. (2015). Investigation of transient MHD Couette 5ow and heat transfer of dusty fluid with temperature-dependent properties. *Journal of Applied Fluid Mechanics*, 8(4), 921–929.

Narahari, M. (2012). Transient free convection flow between long vertical parallel plates with ramped wall temperature at one boundary in the presence of thermal radiation and constant mass diffusion. *Meccanica*, 47(8), 1961–1976.

Ojemeri G. and Onwubuya I. O. (2023). Significance of viscous dissipation and porosity effects in a heated superhydrophobic microchannel. *J. of Engineering and Technology (JET)*, 14(2), 1-19.

Omokhuale, E. and Dange, M. S. (2023). Natural convection Couette flow in the presence of magnetic field and thermal property. *Int. J. of Science for Global Sustainability*, 9(2), 10 – 20. DOI: 10.57233/ijsgs.v9i2.453

Omokhuale, E. and Ojemeri, G. (2024). Couette flow in the presence of viscous dissipative fluid along an upstanding channel affected by Newtonian heating: homotopy perturbation approach. *Journal of Basic Physical Research*, 12(1), 1-12.

Omokhuale, E., Ojemeri, G., Muhammad, A. and Usman, H. (2024a). Significance of viscous dissipation on hydromagnetic oscillatory flow affected by nanoparticles through a boundary layer regime. *International Journal of Development Mathematics*, 1(1), 098-113.

Omokhuale, E., Abubakar, J. and Ojemeri, G. (2024b). Mixed convection flow of a magnetized nanofluid with viscous dissipation in an oscillatory system, *International Journal of Science for Global Sustainability*, 10(2), 229-241. https://doi.org/10.57233/ijsgs.v10i2.672

Omokhuale, E. and Jabaka, M. L. (2022a). Unsteady Convective Couette flow with heat sink and radiation effects. *FUDMA J. of Sciences*, 6(1), 266 – 277. DOI: 10.33003/fjs-20220601-873

Omokhuale, E. and Jabaka, M. L. (2022b). Unsteady convective Couette flow with heat source. *Int. J. of Science for Global Sustainability*, 8(1), 25 – 37. DOI: 10.57233/ijsgs.v8i1.317.

Onwubuya, I. O., Ojemeri, G. and Uko, M. A. (2024). Performance assessment of viscous dissipative fluid due to heat source/sink in a slit microchannel, *International Journal of Science for Global Sustainability,* 10(3), 139-151. https://doi.org/10.57233/ijsgs.v10i3.710

Prakash, D. and Sivakumar, N. (2018). Influence of viscous and holmic heating on MHD flow of nanofluid over an inclined nonlinear stretching sheet embedded in a porous medium, *Int. J. of Mechanical Engineering and Technology (IJM ET),* 9(8), pp. 992-1001.

Qayyum, S., Hayat, T., Shehzad, S. A. and Alsaedi, A. (2017). Effect of a chemical reaction on magnetohydrodynamic (MHD) stagnation point flow of Walters-b nanofluid with Newtonian heat and mass conditions. *Nuclear Engineering and Technology*, 49(8), 1636–1644.

Rajput, U. and Sahu, P. (2011a). Combined effects of chemical reactions and heat generation/absorption on unsteady transient free convection MHD flow between two long vertical parallel plates through a porous medium with constant temperature and mass diffusion. *Elixir Appl. Math.*, 39, 4855–4859.

Rajput, U. and Sahu, P. (2011b). Transient free convection MHD flow between two long vertical parallel plates with constant temperature and variable mass diffusion. *Journal of Math. Analysis*, 34(5), 1665–1671.

Raju, R. S., Jithender Reddy, G. Rao, J. A. and Rashidi, M. M. (2016): thermal diffusion and diffusion thermo effects on an unsteady heat and mass transfer magneto-hydrodynamic natural convection Couette flow using FEM. *Journal of Computational Design and Engr.*, 3(4), 349–362.

Raptis, A. (1998). Flow of microplolar fluid past a continuously moving plate by the presence of radiation. *International Journal of Heat and Mass Transfer*, 41, 2865-2866.

Raptis, A. (1999). Radiation and viscoelastic flow. *International Journal of Heat and Mass Transfer*, 26, 889-895.

Shehzad, S., Hayat, T., Alhuthali, M. and Asghar, S. (2014). MHD three-dimensional flow of Jeffrey fluid with Newtonian heating. *Journal of Central South University*, 21(4), 1428– 1433.

Singh, A. and Sarveshanand, M. (2015). Magnetohydrodynamic free convection between vertical parallel porous plates in the presence of induced magnetic field. *SpringerPlus*, 4(1), 333.

Swain, B. K., Parida, C., Kar, S. and Senapati, N. (2020). Viscous dissipation and joule heating effect on MHD flow and heat transfer past a stretching sheet embedded in a porous medium. *Heliyon*, e05338, 1-8.

Uka, U. A., Emeziem, I. C., Ayinde, S. A., Adenika, C. and Agbo, K. O. (2023). Enhancement of heat transfer with viscous dissipation impact on fluid flow past a moving wedge in a permeable domain. *Eskişehir Technical University J. of Sci. and Techn. A- Applied Sci. and Engr.*, 24(3), 164-176. DOI: 10.18038/estubtda.1197024

Yale, I. D., Uchiri, A. M. T., Hamza, M. M. and Ojemeri, G. (2023). Effect of viscous dissipation fluid in a slit microchannel with heated superhydrophobic surface. *Dutse Journal of Pure and Applied Sciences*, 9(3b), 290-302.

Zin, N. A. M., Khan, I. and Shafie, S. (2018). Exact and numerical solutions for unsteady heat and mass transfer problem of Jeffrey fluid with MHD and Newtonian heating effects. *Neural Computing and Applications*, 30(11), 3491–3507.

Zulkiflee, F., Mohamad, A. Q., Shafie, S. and Khan, A. (2019). Unsteady free convection flow between two vertical parallel plates with Newtonian heating. *Matematika MJIAM*, 35(2), 117–127.

APPENDIX

$$a_1=\frac{1}{1+R}, a_2=\frac{Gr^3a_1^3L_1^2}{120}, a_3=\frac{Gr^3a_1^3L_1L_2}{20}, a_4=\frac{Gra_1^2L_1L_3}{12}, a_5=\frac{Gra_1^2L_2L_3}{3}, a_6=Gr^2a_1^3L_2^2,$$

$$a_7=a_1\frac{L_2^2}{2}, a_{14}=\frac{Gr^3a_1^3L_1^2}{6720}, a_{15}=\frac{Gr^3a_1^3L_1L_2}{840}, a_{16}=\frac{Gra_1^2L_1L_3}{12}, a_{17}=\frac{Gra_1^2L_2L_3}{3}, a_{18}=Gr^2a_1^3L_2^2/360. a_{19}=$$

$$\frac{Gra_1L_3^2}{24}, a_{20}=\frac{GrL_5}{6}, a_{21}=\frac{GrL_6}{2}, L_1=-\frac{\gamma}{a_1(1-\gamma)}, L_2=-L_1, L_3=-Gr\left(\frac{a_1L_1}{6}+\frac{a_1L_2}{2}\right)+1,$$

$$L_4=-1, L_5=-\gamma L_6, L_6=\frac{(a_2+a_3+a_4+a_5+a_6+a_7)}{(1-\gamma)}, L_7=-a_{14}-a_{15}-a_{16}-a_{17}-a_{18}+a_{19}+a_{20}$$

$$L_8=0$$

NOMENCLATURE

B_0 = Constant magnetic flux density [kg/s^2.m^2]
g=Gravitational acceleration [m/s^2]
h= Width of the channel [m]
C_pC_v=Specific heats at constant pressure and constant volume [Jkg^{-1}K^{-1}]
γ = Navier slip parameter
Br = Viscous Dissipation parameter
R = Thermal Radiation parameter
Nu=Dimensionless heat transfer rate
T=Dimensionless temperature of the fluid [K]
T_0=Reference temperature [K]
u=Dimensionless velocity of the fluid [ms^{-1}]
y= Dimensionless distance between plates
U_0=Reference velocity [ms^{-1}]
q_r^* = The radiative heat flux [Wm^{-2}]
K^* = Roseland mean absorption parameter [m^{-1}]

GREEK LETTERS

β =Thermal expansion coefficient [K^{-1}]
μ =Variable fluid viscosity [kgm^{-1}s^{-1}]
k=Thermal conductivity [m.kg/s^3.K]
α =Thermal diffusivity [m^{2}s^{-1}]
γ_s = Ratios of specific heats (C_pC_v)
σ =Electrical conductivity of the fluid [s^{3}m^2/kg]
ρ = Density of the fluid [kgm^{-3}]
v =Fluid kinematic viscosity [m^{2}s^{-1}]
σ^* = The Stefan-Boltzmann constant [W m^{-2} K^{-4}]