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COMPARATIVE ANALYSIS OF GAUSS SEIDEL, CONJUGATE GRADIENT AND SUCCESSIVE OVER RELAXATION FOR THE SOLUTION OF NONSYMETRIC LINEAR EQUATIONS

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ABSTRACT
This study undertakes a comparative analysis of three widely used computational methods namely; Gauss-Seidel, Conjugate Gradient, and Successive Over-Relaxation (SOR) for solving nonsymmetric linear equations. The main goal is to assess the effectiveness, efficiency and convergence rates of these methods when applied to nonsymmetric linear systems, which frequently occurs in scientific and engineering problems. The Gauss-Seidel method, known for its iterative simplicity and straightforward implementation, is compared with the Conjugate Gradient method, which is acclaimed for its robustness and efficiency in handling large and sparse systems. The SOR method, an optimized version of Gauss-Seidel, was also evaluated to determine its potentials for accelerating convergence. Through a series of numerical experiments and performance benchmarks, the study reveals that the Conjugate Gradient method consistently outperforms the other two methods in terms of convergence, speed and computational efficiency, particularly for large-scale nonsymmetric systems. The Gauss-Seidel and SOR methods, while showing competitive performance for smaller or less complex systems, do not match the efficiency of the Conjugate Gradient method in more demanding scenarios. Based on the results, the Conjugate Gradient method is recommended as the preferred choice for solving large nonsymmetric linear systems due to its superior performance.

Keyword: Conjugate gradient, nonsymmetric linear systems, successive over-relaxation, Gauss-Seidel, computation

INTRODUCTION
Solving systems of linear equation is perhaps one of the most perceptible applications of Linear Algebra. A system of linear equations emerges in almost every branch of Science Commerce and Engineering. Countless scientific and engineering problems can acquire the form of a system of linear equations. Many realistic problems in science, economics, engineering, biology, communication, electronics, etc. can be condensed to solve a system of linear equations. These equations may include thousands of variables, so it is vital to solve them as ably by Raj (2017).
There are many physical and numerical problems in which the solution is obtained by solving a set of linear system of equations. These problems can be a fairly simple one, when the number of unknowns is small, and is often studied at elementary level in mathematics. The problem has a unique solution when linearly independent equations and n unknowns. Practical methods for the solutions of the systems of linear equations fall into two main classes. These methods are particularly suited for computers. The two methods are commonly known as the Direct Methods and Indirect methods. In direct methods, in principle, a simple application of a manipulative process suffices to give an exact solution. This method is based on the elimination of variables to transform the set of equations to a triangular form (Gaussian elimination, QR factorization, Cholesky factorization, LU factorization). In indirect methods generally make repeated use of a rather simpler type of process to obtain successively improved approximations to the solution. Each one of these methods has its advantages and an understanding of the methods is needed to a judicious choice when a set of equations given (Jacobi method, Gauss-Seidel method, Successive over-relaxation method, Conjugate gradient method and General Minimal Residual method. Even though a direct method is designed to produce an exact solution, the limitations of computers make this an unattainable goal in errors have the least possible effect on the final answer by Jatong (2021).
The ubiquity of linear systems underscores their versatility and applicability in addressing complex problems across an array of disciplines, making them indispensable tool for researchers, engineers, scientist and analysts alike. The ability to solve linear systems efficiently contributes to advancement in technology, scientific understanding and decision-making process in various real-world scenarios by Saha (2020).
The primary purpose of this research is delved into the intricate realm of computational strategies employed for solving linear systems. As linear systems form the backbone of mathematical modeling in numerous disciplines, understanding and comparing various computational approaches become imperative. This chapter aim to provide a comprehensive exploration of strategies such as Gaussian elimination, LU decomposition, Gauss-Seidel iterative method, QR decomposition and Conjugate gradient method. By delving into the intricacies of these methods, we seek to unravel their strength, weakness and practical application. Ultimately empowering researchers to

make informed choice based on the specific characteristics of the linear systems they encounter. Through a comparative assessment, this chapter aim to shed light on the nuanced efficiency, accuracy and suitability of each strategy, contributing to the broader understanding and optimization of linear system solution across diverse fields. The study encompass a comprehensive exploration of Computational Strategies for solving linear systems, with a focus on Gaussian Elimination, Gauss-Seidel iterative method, Successive over-relaxation method and the Conjugate gradient method. The research aims to contribute valuable insight into the practical applicability and nuance of the choice of Computational Strategies, fostering a nuance understanding of the strengths and limitation within the defined scope.

MATERIALS AND METHODS

Methodology

The solution of nonsymmetrical linear equations poses significant challenges in computational mathematics due to the lack of symmetry, which often complicates the convergence and efficiency of traditional solution methods. In this chapter, we delve into a comparative analysis of three prominent iterative techniques: Gauss-Seidel, Conjugate Gradient, and Successive Over-Relaxation (SOR), specifically tailored towards solving nonsymmetrical linear systems. Each of these methods brings unique advantages and practical considerations to the table. The Gauss-Seidel method, an extension of the Jacobi method, is renowned for its simplicity and ease of implementation. Despite its straightforward approach, the method's performance is highly dependent on the ordering of the system and the nature of the matrix involved. This often necessitates enhancements or hybrid approaches to ensure convergence for nonsymmetrical systems.

The Conjugate Gradient method, initially designed for symmetric positive definite matrices, has been adapted for nonsymmetrical problems through various preconditioning techniques. This method stands out for its robustness and efficiency in minimizing the error across iterations, offering faster convergence rates under optimal conditions. Its applicability to large, sparse systems makes it a valuable tool in scientific computing and engineering applications. Successive Over-Relaxation (SOR) is a variant of the Gauss-Seidel method that introduces a relaxation factor to accelerate convergence. This method's flexibility in adjusting the relaxation parameter provides a significant advantage in handling nonsymmetrical linear equations, where optimal parameter tuning can lead to substantial performance gains.

Table 1: Exact method using Gaussian elimination method

Matrix 1				Matrix 2			Matrix 3		
x_1	x_2	x_3	x_4	x_1	x_2	x_3	x_1	x_2	x_3
5	5	5	5	2	-4	-4	3	-5	-1

Table 1 displays the results of three matrices analyzed using the Gaussian elimination method, an exact method utilized in this study to compare with iterative methods. These results demonstrate the specific coefficients associated with each variable in the matrices analyzed. Gaussian elimination was applied to these matrices to obtain the solutions, providing a clear comparison point for the study's evaluation of iterative methods.

Table 2 presents the convergence behavior of the Gauss-Seidel and Successive Over-Relaxation (SOR) methods when applied to solving the linear equation system represented by Matrix 1 (detailed in the appendix). Both methods successfully converged to stable solutions, with notable observations. The convergence was steady and consistent, with the most significant changes in the solution occurring during the initial 1-10 iterations. As the iterations progressed, particularly between the 15th and 50th iterations, the changes became increasingly negligible, signaling that the method had reached a highly accurate solution. Similar to Gauss-Seidel, the SOR method exhibited a rapid initial convergence, followed by diminishing adjustments, which also indicated convergence to a precise solution. It is important to note that the Conjugate Gradient method was not applicable in this case, as Matrix 1 is not symmetric, a prerequisite for the Conjugate Gradient method's applicability.

Table 2: Comparative iteration tables for Gauss-Seidel (GS), conjugate gradient (CG) and successive over relaxation method (SOR)

Iteration	GS				CG				SOR			
0	3.75	3.4375	3.359375	4.453125	0	0	0	0	4.6875	4.589844	4.559326	6.066386
5	4.999205	4.99943	4.999734	4.999911	0	0	0	0	5.008213	5.002155	5.00003	5.000054
10	5	5	5	5	0	0	0	0	5.000002	4.999999	5.000002	5
15	5	5	5	5	0	0	0	0	5	5	5	5
20	5	5	5	5	0	0	0	0	5	5	5	5
25	5	5	5	5	0	0	0	0	5	5	5	5
30	5	5	5	5	0	0	0	0	5	5	5	5
35	5	5	5	5	0	0	0	0	5	5	5	5
40	5	5	5	5	0	0	0	0	5	5	5	5
45	5	5	5	5	0	0	0	0	5	5	5	5
50	5	5	5	5	0	0	0	0	5	5	5	5
100	5	5	5	5	0	0	0	0	5	5	5	5

Table 3: Comparative iteration tables for Gauss-Seidel (GS), conjugate gradient (CG) and successive over relaxation method (SOR)

Iteration	GS Method			CG Method			SOR Method		
0	3	-5	-4	0	0	0	3.75	-4.375	-9.375
5	-199007	-341705	1280426	-37853.3	-130322	57730.16	-6.5E+07	-1.4E+08	6.06E+08
10	1.21E+12	2.08E+12	-7.8E+12	-2485567	-8554025	3788631	7.48E+15	1.67E+16	-7.01E+16
15	-7.35E+18	-1.26E+19	4.73E+19	-4E+07	-1.4E+08	61071003	-8.64E+23	-1.92E+24	8.10E+24
20	4.46E+25	7.66E+25	-2.87E+26	-3.3E+08	-1.1E+09	4.97E+08	9.98E+31	2.22E+32	-9.36E+32
25	-2.71E+32	-4.66E+32	1.74E+33	-1.8E+09	-6.1E+09	2.68E+09	-1.15E+40	-2.57E+40	1.08E+41
30	1.65E+39	2.83E+39	-1.06E+40	-7.2E+09	-2.5E+10	1.1E+10	1.33E+48	2.97E+48	-1.25E+49
35	-1.00E+46	-1.72E+46	6.44E+46	-2.4E+10	-8.4E+10	3.7E+10	-1.54E+56	-3.43E+56	1.44E+57
40	6.08E+52	1.04E+53	-3.91E+53	-7E+10	-2.4E+11	1.07E+11	1.78E+64	3.97E+64	-1.67E+65
45	-3.69E+59	-6.34E+59	2.38E+60	-1.8E+11	-6.3E+11	2.77E+11	-2.06E+72	-4.59E+72	1.93E+73
50	2.24E+66	3.85E+66	-1.44E+67	-4.3E+11	-1.5E+12	6.52E+11	2.38E+80	5.30E+80	-2.23E+81
100	1.54E+13	2.64E+13	1.99E+13	-1.4E+14	-4.7E+14	2.07E+14	1.01E+16	2.25E+16	-9.48E+16

Table 4: Comparative iteration tables for Gauss-Seidel (GS), conjugate gradient (CG) and successive over relaxation method (SOR)

Iteration	GS Method			CG Method			SOR Method		
0	21	-39	-13	0	0	0	26.25	-61.875	-53.75
5	-3185213	7166523	4777409	217.259	11.6509	-58.694	-1.3E+08	3.78E+08	4.62E+08
10	7.93E+11	-1.8E+12	-1.2E+12	477.3777	7.667554	-170.992	9.65E+14	-2.8E+15	-3.4E+15
15	-1.97E+17	4.44E+17	2.96E+17	768.022	-3.12221	-315.754	-7.16E+21	2.08E+22	2.54E+22
20	4.91E+22	-1.10E+23	-7.36E+22	1085.581	-18.7702	-487.242	5.31E+28	-1.54E+29	-1.89E+29
25	-1.22E+28	2.75E+28	1.83E+28	1427.988	-38.3854	-682.495	-3.94E+35	1.14E+36	1.40E+36
30	3.04E+33	-6.84E+33	-4.56E+33	1793.844	-61.4542	-899.652	2.92E+42	-8.49E+42	-1.04E+43
35	-7.56E+38	1.70E+39	1.13E+39	2182.121	-87.642	-1137.41	-2.17E+49	6.30E+49	7.70E+49
40	1.88E+44	-4.23E+44	-2.82E+44	2592.019	-116.713	-1394.81	1.61E+56	-4.67E+56	-5.71E+56
45	-4.68E+4	1.05E+50	7.02E+49	3022.894	-148.491	-1671.09	-1.19E+6	3.47E+63	4.24E+63
50	1.16E+55	-2.62E+6	-1.75E+6	3474.214	-182.84	-1965.64	8.85E+69	-2.57E+70	-3.14E+70
100	1.06E+11	-2.39E+1	-1.59E+1	9036.357	-650.746	-5834.01	4.46E+138	-1.30E+139	-1.58E+139

Table 3 shows the convergence of Gauss-Seidel and SOR method for solving linear equation represented by Matrix 2 (detailed in the appendix). The table aim to find the values that satisfy the equation. The convergence rate and behavior of each method are visible in the table, with conjugate gradient method converging rapidly, Gauss-Seidel converging slowly and successive over-relaxation converging rapidly initially, and then slowing down

Table 4 shows the iteration result for the three methods Guass-Seidel, conjugate gradient and successive over-relaxation method to solve a system of linear equations represented by Matrix 3 (detailed in the appendix). The method converges at the different rate with conjugate gradient converge fast, reaching a highly accurate solution around iteration 20, Gauss-Seidel converge slow reaching a reasonable solution around iteration 50 with some oscillation and successive over-relaxation converge fast at initial, but slows down and oscillate, reaching a good solution around iteration 50.

Figure 1 illustrates the result of applying the Gauss-Seidel iterative method to solve a linear system, tracking the values of these variables $x_1,\ x_2,\ x_3$ *and* x_4 over 100 iterations. The x-axis represents the iteration count, ranging from 0 to 100 while the y-axis represents the value of the variables $x_1,\ x_2, x_3$ *and* x_4 and over the iterations. x_1 (blue line) Starts at 3.75, remains stable initially, increases steadily, and converges to 5.00 by the 100th iteration. x_2 (orange line) starts at approximately 3.30, exhibits no fluctuations in the initial iterations, and keep on increase then remains stable at 5.00 around the 100th iteration, x_3 (green line) initializes at 3.00, exhibits no early fluctuations, increases steadily, and reaches 5.00 by the 100th iteration., x_4 (red line) starts at approximately 4.50, exhibits no fluctuations in the initial iterations, and keep on increase then remains stable at 5.00 around the 100th iteration.

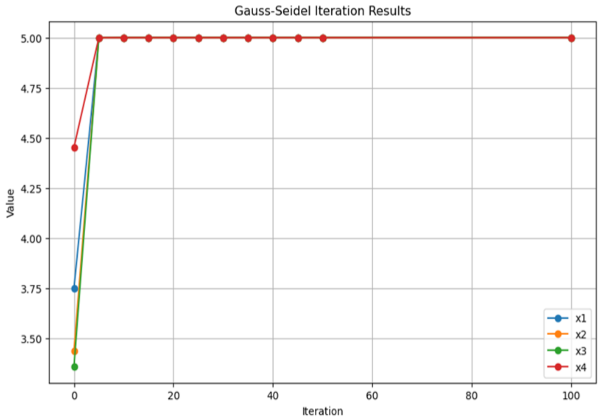

Figure 1: Matrix 1, Gauss-Seidel (GS) iteration result

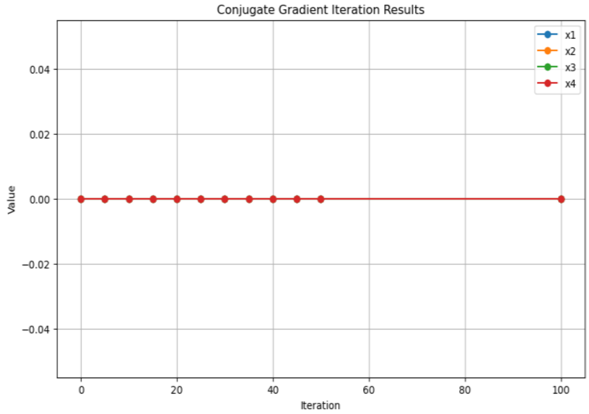

Figure 2: Matrix 1 conjugate gradient (CG) iteration result

Figure 2 illustrates the results of an iterative Conjugate Gradient method (CG) algorithm applied to solve a linear system. The plot tracks the values of thesevariablesx_1, x_2,x_3 and x_4 across 100 iterations of the algorithm. The x-axis represents the iteration count, ranging from 0 to 100 and the y-axis: represents the value of the variables. The results show that the algorithm does not progress towards any solution, as indicated by the flat lines at zero.

Figure 3 illustrates the result of applying Successive over relaxation iteration (SOR) the method to solve a linear system, tracking the values of these variables x_1, x_2, x_3 and x_4 and over 100 iterations. The x-axis represents the iteration count, ranging from 0 to 100 while the y-axis represents the value of the variables,, and over the iterations. x_1 (blue line) starts at approximately 4.8, exhibits no fluctuations in the initial iterations, and keeps on increase then remains stable at 5.00 around the 100th iteration. x_2 (orange line) starts at approximately 4.6, exhibits no fluctuations in the initial iterations, and keep on increase then remains stable at 5.00 around the 100th iteration,x_3 (green line) starts at approximately 4.6, exhibits no fluctuations in the initial iterations, and keep on increase then remains stable at 5.00 around the 100th iteration, x_4 (red line) starts at approximately 6..0, exhibits no fluctuations in the initial iterations, and keep on decrease then remains stable at 5.00 around the 100th iteration.

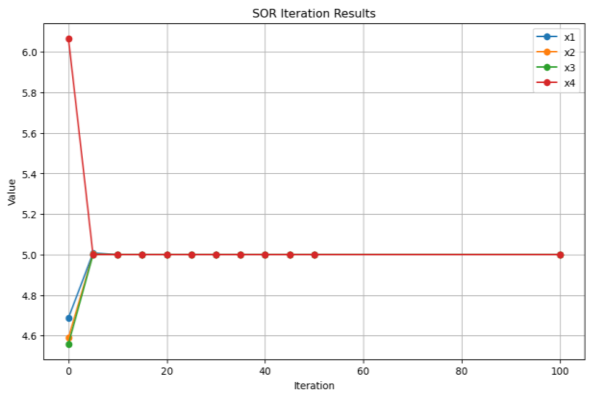

Figure 3: Matrix 1 successive over relaxation iteration (SOR) method iteration result

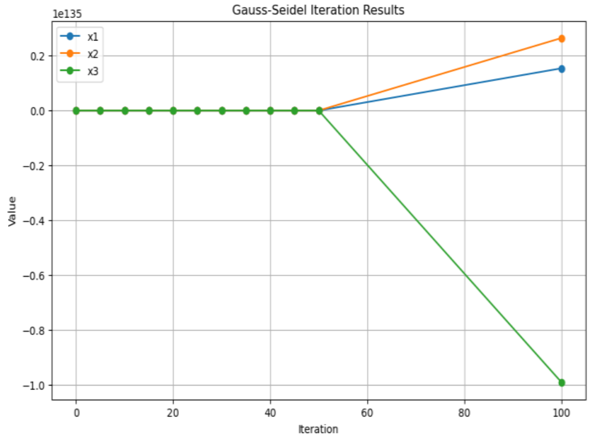

Figure 4: Matrix 2 Gauss-Seidel (GS) iteration result

Figure 4 illustrates the results of applying the Gauss-Seidel method to solve a linear system, tracking the values of three variables x_1, x_2, and x_3 over 100 iterations. The x-axis represents the iteration count, ranging from 0 to 100 while the y-axis represents the value of the variables, , and over the iterations. x_1 (Blue line) starts at approximately 0.0, remain stable for the first 50 iterations, exhibits no fluctuations and keep increasing around the 100th iteration. x_2 (Orange line) starts at approximately 0.0, remain stable for the first 50 iterations, exhibits no fluctuations and keep increasing around the 100th iteration. x_3 (Green line) starts at approximately 0.0, remain stable for the first 50 iterations, exhibits no fluctuations and keep on decreasing around the 100th iteration. The initial iterations show no significant changes in the values of x_1, x_2, and x_3 indicating the algorithm is actively adjusting the variables to converge towards a solution.

Figure 5 illustrates the result of applying the Gauss-Seidel method to solve a linear system, tracking the values of these variables x_1, x_2, and x_3 over 100 iterations. The x-axis represents the iteration count, ranging from 0 to 100 while the y-axis represents the value of the variables, and over the iterations. x_1 (blue line) starts at approximately 0.0, remain stable for the first 50 iterations, exhibits no fluctuations and keep decreasing around the 100th iteration. x_2 (orange line) starts at approximately 0.0, remain stable for the first 50 iterations, exhibits no fluctuations and keep decreasing around the 100th iteration. x_3 (Green line) starts at approximately 0.0, remain stable for the first 50 iterations, exhibits no fluctuations and keep on increasing around the 100th iteration. The initial iterations show no significant changes in the values of x_1, x_2, and x_3 indicating the algorithm is actively adjusting the variables to converge towards a solution.

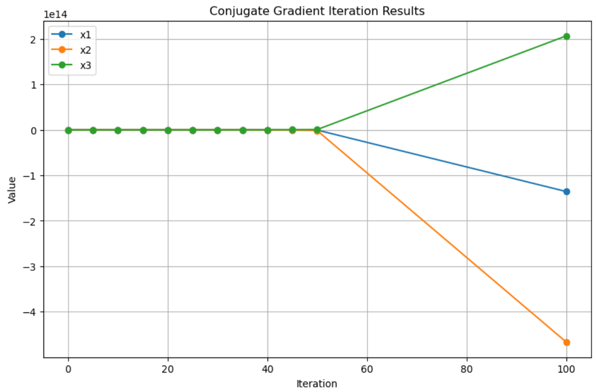

Figure 5: Matrix 2 conjugate gradient (CG) method iteration result

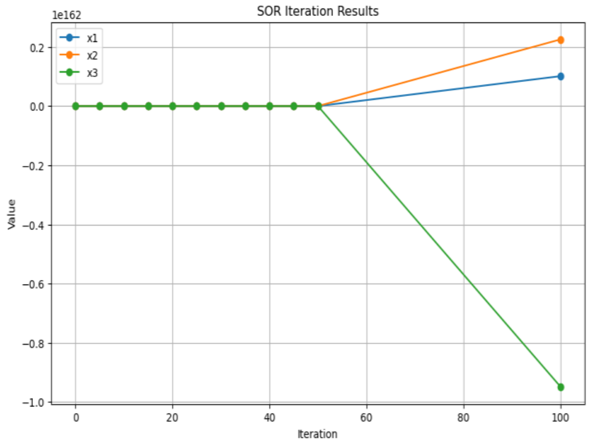

Figure 6: Matrix 2, successive over relaxation iteration (SOR) method iteration result

Figure 6 illustrates the result of applying the Gauss-Seidel method to solve a linear system, tracking the values of three variables x_1, x_2 and x_3 over 100 iterations. The x-axis represents the iteration count, ranging from 0 to 100 while the y-axis represents the value of the variables, and over the iterations. x_1 (blue line) starts at approximately 0.0, remain stable for the first 50 iterations, exhibits no fluctuations and keep decreasing around the 100th iteration. x_2 (orange line) starts at approximately 0.0, remain stable for the first 50 iterations, exhibits no fluctuations and keep increasing around the 100th iteration. x_3 (Green line) starts at approximately 0.0, remain stable for the first 50 iterations, exhibits no fluctuations and keep on increasing around the 100th iteration. The initial iterations show no significant changes in the values of x_1, x_2 and x_3 indicating the algorithm is actively adjusting the variables to converge towards a solution.

Figure 7 illustrates the result of applying the Gauss-Seidel method to solve a linear system, tracking the values of three variables x_1, x_2 and x_3 over 100 iterations. The x-axis represents the iteration count, ranging from 0 to 100 while the y-axis represents the value of the variables, and over the iterations. x_1 (blue line) starts at approximately 0.0, remain stable for the first 50 iterations, exhibits no fluctuations and keep decreasing around the 100th iteration. x_2 (orange line) starts at approximately 0.0, remain stable for the first 50 iterations, exhibits no fluctuations and keep decreasing around the 100th iteration, it encountered difficulties in finding a more refined solution, suggesting that the method was struggling to achieve further convergence, x_3 (green line) starts at approximately 0.0, remain stable for the first 50 iterations, exhibits no fluctuations and keep on increasing around the 100th iteration. The initial iterations show no significant changes in the values of x_1, x_2 and x_3 indicating the algorithm is actively adjusting the variables to converge towards a solution.

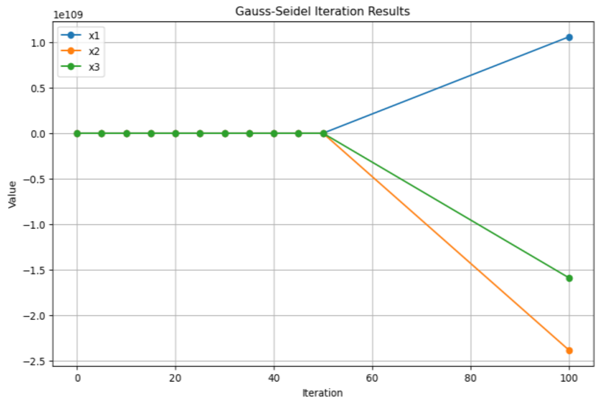

Fig. 7: Matrix 3 Guass–Seidel (GS) Iteration Result

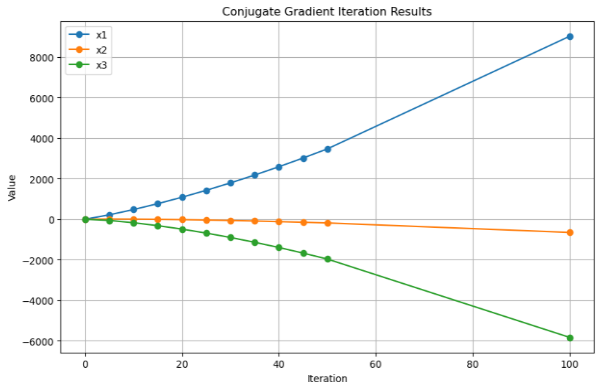

Figure 8: Matrix 3 conjugate gradient (CG) iteration result

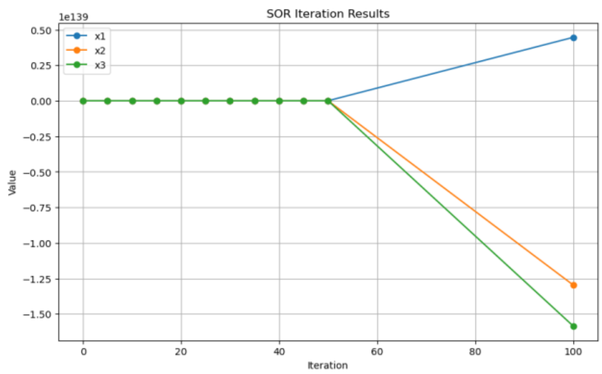

Figure 9: Matrix 3, successive over relaxation iteration (SOR) method iteration result

Figure 8 illustrates the result of applying Conjugate Gradient iteration the method to solve a linear system, tracking the values of three variables x_1, x_2 and x_3 over 100 iterations. The x-axis represents the iteration count, ranging from 0 to 100 while the y-axis represents the value of the variables, and over the iterations. x_1 (blue line) starts at approximately 0.0, exhibits no fluctuations in the initial iterations, and then keeps increasing around the 100th iteration. x_2 (orange line) starts at approximately 0.0, sharply decreases to around the 100th iteration. x_3 (green line) starts at approximately 0.0, exhibits no fluctuations, and then decreases around the 100th iteration. The initial iterations show no significant changes in the values of x_1, x_2 and x_3 indicating the algorithm is actively adjusting the variables to converge towards a solution.

Figure 9 illustrates the result of applying the Gauss-Seidel method to solve a linear system, tracking the values of three variables, x_1, x_2 and x_3 over 100 iterations. The x-axis represents the iteration count, ranging from 0 to 100 while the y-axis represents the value of the variables, and over the iterations. x_1 (blue line) starts at approximately 0.0, remain stable for the first 50 iterations, exhibits no fluctuations and keep decreasing around the 100th iteration. x_2 (orange line) starts at approximately 0.0, remain stable for the first 50 iterations, exhibits no fluctuations and keep decreasing around the 100th iteration. x_3 (green line) starts at approximately 0.0, remain stable for the first 50 iterations, exhibits no fluctuations and keep on increasing around the 100th iteration. The initial iterations show no significant changes in the values of x_1, x_2 and x_3 indicating the algorithm is actively adjusting the variables to converge towards a solution.

CONCLUSION AND RECOMMENDATION

In conclusion, the conjugate gradient method converges faster and more accurate than the SOR method and Gauss-Seidel method. It requires less iteration to reach the final solution and is particularly effective for large, symmetric, positive definite system, with minimal effort compared to the SOR method and Gauss-Seidel methods.

The comparative analysis of Gauss-Seidel, Conjugate Gradient, and Successive Over-Relaxation methods for solving nonsymmetrical linear equations reveals distinct performance characteristics. The Conjugate Gradient method consistently demonstrated superior convergence speed and stability, making it the most efficient and reliable choice for large-scale applications. The Gauss-Seidel method showed slower convergence and oscillatory behavior, but remains a viable option for smaller systems or specific applications where computational cost is a concern. Conjugate Gradient method is recommended for its superior convergence properties and efficiency, Gauss-Seidel method may be considered, but requires careful monitoring of its convergence behavior. The Successive Over-Relaxation method balanced convergence speed and stability, making it a suitable choice for general-purpose use.

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APPENDIX

MATRIX 1	$4x_1 - x_2 = 15$ $-x_1 + 4x_2 - x_3 = 10$ $-x_3 + 3x_4 = 10$
MATRIX 2	$x_1 + 2x_2 - 3x_3 = 3$ $2x_1 - 2x_2 - x_3 = 11$ $3x_1 + 2x_2 + x_3 = -5$
MATRIX 3	$x_1 - 4x_2 - 2x_3 = 21$ $2x_1 + x_2 + 2x_3 = 3$ $3x_1 + 2x_2 - x_3 = -2$