ENGINEERING

ISSN (Print): 24490954
ISSN (Online): 26364972

NEW RESULT ON A FLUID FLOW INDUCED BY A ROTATING MAGNETIC FIELD

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Manuscript Received: 12/08/2018 Accepted: 15/08/2018 Published: December 2018

ABSTRACT
This research paper provides a new result on induced fluid flow in a circular cylinder under the influence of a rotating magnetic field. Maxwell's induction equation and asymptotic technique are used to obtain the new result and it is observed that the convective term which Mofatt neglected in his work is significant in the boundary layer.

Key words: *Induced fluid flow , Maxwell's Induction equation, Rotating Magnetic field, Convective term, Boundary layer*

INTRODUCTION

In a work by Moffatt(1967), he considered the effect of a rotating magnetic field on an infinitely long circular cylinder of conducting fluid. He assumed a low magnetic Reynolds number Rm and obtained an exact expression for the magnetic field in terms of Bessel's function. He found out that the rate of vorticity generation by the Lorentz force was independent of time and that this steady vorticity source produced a rigid body rotation in the interior of the cylinder inside a viscous-magnetic boundary layer.

Nigam (1969) examined a spherical container of a conducting fluid in a rotating magnetic field but his analysis was identical to that of Moffatt. Dahlberg (1972) showed that the rate of vorticity generation in a circular cylinder was steady for all rotation rates of the magnetic field and he then derived a general solution for the vorticity field. The stability of this solution for the case of a slowly rotating field was examined by Richardson (1974) who concluded that instability would occur for every small magnetic field strength.

Sneyd (1978) studied the effects of a rotating magnetic field on the container of conducting fluid. He assumed a small magnetic Reaynold number and the frequency of alternation or rotation was rapid and so the magnetic field was confined in a thin layer on the surface of the container. He used a boundary layer analysis to find the rate of vorticity generation due to Lorentz force and used a perturbation analysis to study the flow induced in a slightly distorted circular cylinder by a rotating field. His results showed the development of complex flow in the viscous-magnetic boundary layer which may be unstable.

Paul and Singh (1998) have shown the analytical solution of laminar and have fully developed free convective flow between two coaxial vertical cylinders partially filled with a porous medium and clear fluid. Yan (2000) has considered viscous flow about a submerged circular cylinder, which oscillates with steady current. Several solutions in the case of hydromagnetic free-convective flows have been obtained by Chandran *et al.,* (1993; 1996; 1998; 2001), for different physical situations of flow formations. Stiller and Franar (2006), studied early stage of turbulent flow driven by a rotating magnetic field via direct simulation and electric potential measurement for the case of a cylindrical geometry. The numerical results show that the undisturbed flow remain stable up to the linear stability limit whereas small perturbation may initiate nonlinear transition at subcritical Taylor numbers

This work extends the work of Moffatt (1974), by providing a new result and also shows that the convection term which [4] neglected in his work is not negligible in the boundary layer.

MATERIALS AND METHODS

The Maxwell induction equation used in this work is:

$$\frac{\partial \underline{B}}{\partial t} = \nabla \times (\underline{u} \times \underline{B}) + \eta \nabla^2 \underline{B} \quad \text{.............................(2.1)}$$

where $\underline{B}$ is the rotating magnetic field

$\underline{u}$ is the velocity of the moving fluid

η is the magnetic diffusivity of the fluid

t is the time

∇ is the gradient operator.

Let

$$\underline{u} = (0,\ u_\theta,\ 0)$$

$$\underline{B} = (\underline{B}_r,\ 0,\ 0)$$

$$\nabla \times (\underline{u} \times \underline{B}) = -\frac{e_r}{r}\frac{\partial}{\partial \theta}(u_\theta \underline{B}_r) - e_\theta \frac{\partial}{\partial r}(u_\theta \underline{B}_r) \quad .(2.2)$$

Using $\underline{B} = (\underline{B}_r,\ \underline{B}_\theta,\ \underline{B}_z)$, then (2.1) becomes

$$\frac{\partial}{\partial t}(\underline{B}_r,\ \underline{B}_\theta,\ \underline{B}_z) = \left(-\frac{1}{r}\frac{\partial}{\partial \theta}(u_\theta \underline{B}_r) - \frac{\partial}{\partial r}(u_\theta \underline{B}_r),\ 0\right) + (\eta\nabla^2 \underline{B}_r, \eta\nabla^2 \underline{B}_\theta, \eta\nabla^2 \underline{B}_z) \quad (2.3)$$

Then considering the r-component in (2.3) we have

$$\frac{\partial \underline{B}_r}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial \theta}(u_\theta \underline{B}_r) + \eta\nabla^2 \underline{B}_r \quad \text{...................(2.4)}$$

Then using the dimensionless variables for parameters in (2.4), we have

$$\underline{B}_r = \underline{B}'_r \underline{B}_0,\quad u_\theta = u'_\theta v,\quad x = x'L$$

$$r = r' r_0 \text{ and } \theta = \theta' \theta_0$$

$$\frac{\partial}{\partial r} = \frac{1}{r_0}\frac{\partial}{\partial r'}$$

$$\frac{\partial}{\partial \theta} = \frac{1}{\pi}\frac{\partial}{\partial \theta'} \quad \text{......(2.5)}$$

Substituting (2.5) in (2.4) we obtain

$$\frac{L}{v r_0}\frac{\partial \underline{B}_r}{\partial t} = -\frac{L}{\pi r_0 r}\frac{\partial}{\partial \theta}(u_\theta \underline{B}_r) + \frac{1}{R_m}\nabla^2 \underline{B}_r \quad \text{....(2.6)}$$

Let $\frac{a}{R_m} = \frac{L}{v r_0}$ and $b = \frac{L}{\pi r_0}$

Then (2.6) becomes

$$\frac{a}{R_m}\frac{\partial \underline{B}_r}{\partial t} = -\frac{b}{r}\frac{\partial}{\partial \theta}(u_\theta \underline{B}_r) + \frac{1}{R_m}\nabla^2 \underline{B}_r \quad \text{.......(2.7)}$$

Method of Solution

Using asymptotic expansion for $\underline{B}_r$ and u_θ we have

$$\underline{B}_r = \underline{B}_{r_0} + R_m \underline{B}_{r_1} + R_m^2 \underline{B}_{r_2} + R_m^3 \underline{B}_{r_3} + \ldots\ldots$$
$$u_\theta = u_{\theta_0} + R_m u_{\theta_1} + R_m^2 u_{\theta_2} + R_m^3 u_{\theta_3} + \ldots\ldots \quad ..(2.8)$$

Substituting (2.8) in (2.7) and equating the coefficients of $\frac{1}{R_m}$ and the constant term we obtain respectively.

$$a\frac{\partial \underline{B}_{r_0}}{\partial t} = \nabla^2 \underline{B}_{r_0} \quad .....(2.9)$$

and

$$a\frac{\partial \underline{B}_{r_1}}{\partial t} = -\frac{b}{r}\left(u_{\theta_0} \cdot \underline{B}_{r_0}\right) + \nabla^2 \underline{B}_{r_1} \quad ......(2.10)$$

Equation (2.9) was solved by [4] and he obtained a unique solution for $\underline{B}_{r_0}$ but he neglected the convection term $\nabla \times (\underline{u} \times B)$ which resulted in equation (2.10). This paper now extends the work of [4] by solving for $\underline{B}_{r_1}$ in equation (2.10).

RESULTS AND DISCUSSION

Theorem 4.1

Consider the equation

$$\frac{\partial B_{r_1}}{\partial t} = \lambda \frac{\partial^2 B_{r_1}}{\partial t^2} - \frac{\lambda b}{r}\frac{\partial}{\partial \theta}\left(u_{\theta_0} \cdot \underline{B}_{r_0}\right) \qquad r > a \quad .....(4.1.1)$$

$\lambda = \frac{1}{a}$, together with $\underline{B}_{r_1}(r,\ 0) = 0,\ \underline{B}_{r_1}(a,\ t) = 0,\ \underline{B}_{r_1}(\infty,\ t) = 0,$

Then $\underline{B}_{r_1}$ has a unique solution

Proof

Let $f(r,\ t) = -\frac{\lambda b}{r}\frac{\partial}{\partial \theta}\left(u_{\theta_0} \cdot \underline{B}_{r_0}\right)$

Then (4.1.1) becomes

$$\frac{\partial B_{r_1}}{\partial t} - \lambda \frac{\partial^2 B_{r_1}}{\partial r^2} = f(r, \tau), \qquad r > 0, \qquad t > 0$$

whose solution is:

$$B_{r_1} = \int_0^t \int_0^\infty \left[G(r-\xi,\ t-\tau) - G(r-\xi,\ t-\tau)\right] f(\xi,\ \tau)\, d\xi\, d\tau \quad .(4.1.2)$$

where

$$G(r+\xi,\ t-\tau) = \frac{\exp\left[-(r+\xi)^2 / 4\lambda(t-\tau)\right]}{2\sqrt{\pi\lambda(t-\tau)}}$$

and $$G(r-\xi,\ t-\tau) = \frac{\exp\left[-(r-\xi)^2 / 4\lambda(t-\tau)\right]}{2\sqrt{\pi\lambda(t-\tau)}}$$

Hence, the solution is unique and the proof is complete.

Theorem 4.2

B_{r_1} is positive

Proof

Recall $f(r) = e^{i(\theta - \omega t)}\left[B_0\left(\frac{r^2 - a^2}{r}\right)\sin(\theta - \omega t) + \frac{aB_0}{kr}\sqrt{2}\sin\left(\theta - \omega t + \frac{\pi}{4}\right)\right]$ (from [4])

Then substitute for $f(\xi, \tau)$ in (4.1.2) we have:

$$B_{r_1} > \int_0^t \int_0^\infty \left[G(r+\xi,\ t-\tau) - G(r-\xi,\ t-\tau)\right] \times$$
$$e^{i(\theta - \omega t)}\left[B_0\left(\frac{\xi^2 - a^2}{\xi}\right)\sin(\theta - \omega\tau) + \frac{aB_0}{K\xi}\sqrt{2}\sin\left(\theta - \omega t + \frac{\pi}{4}\right)\right] d\xi\, d\tau > 0 \quad (4.2.1)$$

This completes the proof. Hence $\nabla \times (\underline{u} \times \underline{B})$ is not negligible in the boundary layer near $r = a$.

CONCLUSION

The results extends the work of [4] by obtaining Br_1 which was not in [4] and showing that there exists a unique solution for the stated problem and that the convection term $\nabla \times (\underline{u} \times \underline{B})$ which [4] neglected in his research work is not negligible in the boundary layer near $r = a$

ACKNOWLEDGEMENT

The authors appreciate the comments of the reviewers in improving the quality of the paper.

Conflict of Interest

The authors declare that there is no conflict of interest

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